CHAPTER 1

Functions

DPP 1.1

Function: Definition, Domain and Range

Single Correct Answer Type

1. $f(x) = \sin[x] + [\sin x], 0 < x < \frac{\pi}{2}$ (where [.] represents the greatest integer function), can also be represented as

(a) $\begin{cases} 0, & 0 < x < 1 \\ 1 + \sin 1, & 1 \le x < \frac{\pi}{2} \end{cases}$

(b) $\begin{cases} \frac{1}{\sqrt{2}}, & 0 < x < \frac{\pi}{4} \\ 1 + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}, & \frac{\pi}{4} \le x < \frac{\pi}{2} \end{cases}$

(c) $\begin{cases} 0, & 0 < x < 1 \\ \sin 1, & 1 \le x < \frac{\pi}{2} \end{cases}$

(d) $\begin{cases} 0, & 0 < x < \frac{\pi}{4} \\ 1, & \frac{\pi}{4} < x < 1 \\ \sin 1, & 1 \le x < \frac{\pi}{2} \end{cases}$

2. If $\phi(x) = \frac{1}{1+e^{-x}}$ and $S = \phi(5) + \phi(4) + \phi(3) + \ldots + \phi(-3) + \phi(-4) + \phi(-5)$, then the value of S is,

(a) 5 (b) 11/2 (c) 6 (d) 13/2

3. For non-negative integers m and n a function is defined as follows:

$$f(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ f(m-1,1) & \text{if } m \neq 0, n = 0 \\ f(m-1, f(m,n-1)) & \text{if } m \neq 0, n \neq 0 \end{cases}$$

Then the value of $f(1, 1)$ is

(a) 1 (b) 2 (c) 3 (d) 4

4. If $f : R \to Q$ (Rational numbers), $g : R \to Q$ (Rational numbers) are two continuous functions such that $\sqrt{3}f(x) + g(x) = 4$, then $(1 - f(x))^3 + (g(x) - 3)^3$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

5. If f and g are two functions defined on N, such that

$$f(n) = \begin{cases} 2n-1 & \text{if } n \text{ is even} \\ 2n+2 & \text{if } n \text{ is odd} \end{cases} \quad \text{and } g(n) = f(n) + f(n+1).$$

Then range of g is

(a) $\{m \in N : m = \text{multiple of } 4\}$

(b) {set of even natural numbers}

(c) $\{m \in N : m = 4k + 3, k \text{ is a natural number}\}$

(d) $\{m \in N : m = \text{multiple of 3 or multiple of 4}\}$

Cengage Learning India Pvt. Ltd.

6. The number of points on the real line where the function $f(x) = \log_{|x^2-1|} |x-3|$ is not defined is
(a) 4 (b) 5 (c) 6 (d) 7

7. For relation $2\log y - \log x - \log(y-1)$
(a) domain = $[4, +\infty)$, range = $(1, +\infty)$
(b) domain = $[4, +\infty)$, range = $(2, +\infty)$
(c) domain = $(2, +\infty)$, range = $(2, +\infty)$
(d) none of these

8. The range of the function $y = [x^2] - [x]^2, x \in [0, 2]$ (where [.] denotes the greatest integer function), is
(a) $\{0\}$ (b) $\{0, 1\}$
(c) $\{1, 2\}$ (d) $\{0, 1, 2\}$

9. The number of elements in the domain of the function $f(x) = \sin^{-1}\left(\frac{x^2-2x}{3}\right) + \sqrt{[x]+[-x]}$, (where [.] denotes the greatest integer function) is equal to
(a) 6 (b) 4 (c) 3 (d) 5

10. The range of the function $f(x) = \sqrt{\cos^{-1}(\sqrt{\log_4 x}) - \frac{\pi}{2}} + \sin^{-1}\left(\frac{1+x^2}{4x}\right)$ is equal to
(a) $\left(0, \frac{\pi}{2} + \sqrt{\frac{\pi}{2}}\right]$ (b) $\left[\frac{\pi}{2}, \frac{\pi}{2} + \sqrt{\frac{\pi}{2}}\right]$
(c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) $\left\{\frac{\pi}{6}\right\}$

11. The range of the function $f(x) = \tan^{-1}\left(\frac{x^2+1}{x^2+\sqrt{3}}\right), x \in R$ is
(a) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right)$ (b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right)$
(c) $\left[\frac{\pi}{6}, \frac{\pi}{4}\right)$ (d) None of these

12. The domain of the function $f(x) = \sqrt{10 - \sqrt{x^4 - 21x^2}}$ is
(a) $[5, \infty)$
(b) $[-\sqrt{21}, \sqrt{21}]$
(c) $[-5, -\sqrt{21}] \cup [\sqrt{21}, 5] \cup \{0\}$
(d) $(-\infty, -5)$

13. Number of integers in domain of function $f(x) = \log_{[x^2]}(4 - |x|) + \log_2\{\sqrt{x}\}$ (where [.] is G.I.F and {.} is fractional part function of x) is
(a) 0 (b) 1 (c) 2 (d) 3

14. The domain of the function $f(x) = \log_e\{\text{sgn}(9 - x^2)\} + \sqrt{[x]^3 - 4[x]}$ (where [.] represents the greatest integer function is
(a) $[-2, 1) \cup [2, 3)$ (b) $[-4, 1) \cup [2, 3)$
(c) $[4, 1) \cup [2, 3)$ (d) $[2, 1) \cup [2, 3)$

15. Number of integers in the range of $f(x) = \frac{1}{\pi}(\sin^{-1} x + \tan^{-1} x) + \frac{x+1}{x^2+2x+5}$ is
(a) 0 (b) 3 (c) 2 (d) 1

Multiple Correct Answers Type

16. The function $f(x) = \cos^{-1}\left(\frac{2[|\sin x| + |\cos x|]}{\sin^2 x + 2\sin x + \frac{11}{4}}\right)$ is defined if x belongs to (where [.] represents the greatest integer function)
(a) $\left[0, \frac{7\pi}{6}\right]$ (b) $\left[0, \frac{\pi}{6}\right]$
(c) $\left[\frac{11\pi}{6}, 2\pi\right]$ (d) $[\pi, 2\pi]$

17. Which of the following functions are defined for all x?
(a) $\sin[x] + \cos[x]$ ([x] denotes the greatest integer $\le x$)
(b) $\sec^{-1}(1 + \sin^2 x)$
(c) $\tan(\log x)$
(d) $\sqrt{\frac{9}{8} + \cos x + \cos 2x}$

18. Which of the following functions is/are bounded?
(a) $f(x) = \frac{2x}{1+x^2}, [-2, 2]$
(b) $f(x) = \frac{x^2}{1-x}, x \in [0, 2] - \{1\}$
(c) $f(x) = \frac{x^3 - 8x + 6}{4x+1}, [0, 5]$
(d) none of these

Answers Key

Single Correct Answer Type

1. (c)	2. (b)	3. (c)	4. (b)	5. (c)
6. (c)	7. (a)	8. (d)	9. (d)	10. (d)
11. (c)	12. (c)	13. (c)	14. (a)	15. (c)

Multiple Correct Answers Type

16. (a, b, c) 17. (a, b, d)
18. (a, c)

Cengage Learning India Pvt. Ltd.

DPP 1.2

Odd and Even Functions, Periodic Functions

Single Correct Answer Type

1. Let $G(x)=\left(\frac{1}{a^x-1}+\frac{1}{2}\right)F(x)$, where a is a positive real number not equal to 1 and $F(x)$ is an odd function. Which of the following statements is true?
(a) $G(x)$ is an odd function
(b) $G(x)$ is an even function
(c) $G(x)$ is neither even function nor odd function
(d) Whether $G(x)$ is an odd function or an even function, it depends on the value of a

2. Let $f(x) = ([a]^2 - 5[a] + 4)x^3 - (6\{a\}^2 - 5\{a\} + 1)\,x - (\tan x)\,\text{sgn}\,x$, be an even function for all $x \in R$, then sum of all possible values of 'a' is (where [ ] and { } denote greatest integer function and fractional part functions respectively)
(a) $\frac{17}{6}$ (b) $\frac{53}{6}$ (c) $\frac{31}{3}$ (d) $\frac{35}{3}$

3. Let $f(x) = \sin^2 x + \cos^4 x + 2$ and $g(x) = \cos(\cos x) + \cos(\sin x)$. Also let period of $f(x)$ and $g(x)$ be T_1 and T_2 respectively then
(a) $T_1 = 2T_2$ (b) $2T_1 = T_2$
(c) $T_1 = T_2$ (d) $T_1 = 4T_2$

4. A continuous, even periodic function f with period 8 is such that $f(0) = 0, f(1) = -2, f(2) = 1, f(3) = 2, f(4) = 3$, then the value of $\tan^{-1}\tan\{f(-5) + f(20) + \cos^{-1}(f(-10)) + f(17)\}$ is equal to
(a) $2\pi - 3$ (b) $3 - 2\pi$
(c) $2\pi + 3$ (d) $3 - \pi$

5. If a and b are natural numbers and $f(x) = \sin(\sqrt{a^2-3})x + \cos(\sqrt{b^2+7})x$ is periodic with finite fundamental period, then period of $f(x)$ is:
(a) π
(b) 2π
(c) $2\pi(\sqrt{a^2-3}+\sqrt{b^2+7})$
(d) $\pi(\sqrt{a^2-3}+\sqrt{b^2+7})$

6. Period of $f(x) = \sin 3x \cos[3x] - \cos 3x \sin[3x]$ (where [ ] denotes the greatest integer function), is
(a) 1/6 (b) 2/3 (c) 5/6 (d) 1/3

7. What is the fundamental period of $f(x) = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$?
(a) $\pi/2$ (b) π (c) 2π (d) 3π

8. If $f: R \to R$ is a function satisfying the property $f(x + 1) + f(x + 3) = 2$ for all $x \in R$ than f is
(a) periodic with period 3
(b) periodic with period 4
(c) non periodic
(d) periodic with period 5

9. Period of $f(x) = \text{sgn}([x] + [-x])$ is equal to (where [.] denotes greatest integer function)
(a) 1 (b) 2
(c) 3 (d) does not exist

10. If $F(x)$ and $G(x)$ are even and odd extensions of the functions $f(x) = x|x| + \sin|x| + xe^x$, where $x \in (0, 1)$, $g(x) = \cos|x| + x^2 - x$, where $x \in (0, 1)$ respectively to the interval $(-1, 0)$ then $F(x) + G(x)$ in $(-1, 0)$ is
(a) $\sin x + \cos x + xe^{-x}$
(b) $-(\sin x + \cos x + xe^{-x})$
(c) $-(\sin x + \cos x + x + xe^{-x})$
(d) $-(\sin x + \cos x + x^2 + xe^{-x})$

11. Let $P(x) = x^{10} + a_2x^8 + a_3x^6 + a_4x^4 + a_5x^2$ be a polynomial with real coefficients. If $P(1) = 1$ and $P(2) = -5$, then the minimum number of distinct real zeroes of $P(x)$ is
(a) 5 (b) 6 (c) 7 (d) 8

Multiple Correct Answers Type

12. The function 'g' defined by
$$g(x) = \sin(\sin^{-1}\sqrt{\{x\}}) + \cos(\sin^{-1}\sqrt{\{x\}}) - 1$$
(where $\{x\}$ denotes the functional part function) is
(a) an even function
(b) a periodic function
(c) an odd function
(d) neither even nor odd

13. Let f be a differential function such that $f(x) = f(2 - x)$ and $g(x) = f(1 + x)$ then
(a) $g(x)$ is an odd function
(b) $g(x)$ is an even function
(c) graph of $f(x)$ is symmetrical about the line $x = 1$
(d) $f'(1) = 0$

Answers Key

Single Correct Answer Type

1. (b) **2.** (d) **3.** (c) **4.** (d) **5.** (b)
6. (d) **7.** (b) **8.** (b) **9.** (a) **10.** (c)
11. (a)

Multiple Correct Answers Type

12. (a, b) **13.** (b, c, d)

Cengage Learning India Pvt. L

DPP 1.3

Different Types of Mapping

Single Correct Answer Type

1. Let $f : R \to [1, \infty)$ be defined as $f(x) = \log_{10}(\sqrt{3x^2 - 4x + k + 1} + 10)$. If $f(x)$ is surjective, then

(a) $k = \frac{1}{3}$ (b) $k < \frac{1}{3}$

(c) $k > \frac{1}{3}$ (d) $k = 1$

2. Let $f : (-\infty, 0] \to [1, \infty)$ be defined as $f(x) = (1 + \sqrt{-x}) - (\sqrt{-x} - x)$, then $f(x)$ is

(a) injective but not surjective
(b) injective as well as surjective
(c) neither injective nor surjective
(d) surjective but not injective

3. Let A be a non-empty set of real numbers and $f : A \to A$ be such that $f(f(x)) = x$, $\forall\, x \in R$. Then $f(x)$ is

(a) a bijection
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

4. $f : R \to R$ defined by $f(x) = \frac{1}{2} x|x| + \cos x + 1$ is

(a) one-one and onto (b) one-one and into
(c) many-one and onto (d) many-one and into

5. Let $f: R \to R$, $f(x) = \begin{cases} 2x + \alpha^2, & x \geq 2 \\ \frac{\alpha x}{2} + 10, & x < 2 \end{cases}$. If $f(x)$ is onto function, then α belongs to

(a) $[1, 4]$ (b) $[-2, 3]$
(c) $(0, 3]$ (d) $[2, 5]$

6. Let $f: R \to \left[0, \frac{2\pi}{3}\right]$ defined as $f(x) = \cot^{-1}(x^2 - 4x + \alpha)$. Then the smallest integral value of α such that $f(x)$ is into function is

(a) 2 (b) 4 (c) 6 (d) 8

7. Let $f: R^+ \to \{-1, 0, 1\}$ defined by $f(x) = \text{sgn}(x - x^4 + x^7 - x^8 - 1)$ where sgn denotes signum function. Then $f(x)$ is

(a) many-one and onto (b) many-one and into
(c) one-one and onto (d) one-one and into

8. Which of the following statements are incorrect?

I. If $f(x)$ and $g(x)$ are one-one then $f(x) + g(x)$ is also one-one.
II. If $f(x)$ and $g(x)$ are one-one then $f(x) \cdot g(x)$ is also one-one.
III. If $f(x)$ is odd then it is necessarily one-one.

(a) I and II only (b) II and III only
(c) III and I only (d) I, II and III

9. Which of the following functions is one-one?

(a) $f : R \to R$ defined as $f(x) = e^{3\text{sgn}\, x} + e^{x^2}$

(b) $f : [-1, \infty) \to (0, \infty)$ defined by $f(x) = e^{x^2 + |x|}$

(c) $f : [3, 4] \to [4, 6]$ defined by $f(x) = |x - 1| + |x - 2| + |x - 3| + |x - 4|$

(d) $f(x) = \sqrt{\ln(\cos(\sin x))}$

Comprehension Type

For Questions 10–12

Let $f(x) = x^2 - 2x - 1$ $\forall\, x \in R$. Let $f : (-\infty, a] \to [b, \infty)$, where a is the largest real number for which $f(x)$ is bijective.

10. If $f : R \to R$, $g(x) = f(x) + 3x - 1$, then the least value of function $y = g(|x|)$ is

(a) $-9/4$ (b) $-5/4$ (c) -2 (d) -1

11. If $f : [a, \infty) \to [b, \infty)$, then $f^{-1}(x)$ is given by

(a) $1 + \sqrt{x + 2}$ (b) $1 - \sqrt{x + 3}$
(c) $1 - \sqrt{x + 2}$ (d) $1 + \sqrt{x + 3}$

12. If $f : R \to R$, then range of values of k for which equation $f(|x|) = k$ has 4 distinct real roots is

(a) $(-2, -1)$ (b) $(-2, 0)$
(c) $(-1, 0)$ (d) $(0, 1)$

Answers Key

gle Correct Answer Type

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (c)
6. (b) **7.** (b) **8.** (d) **9.** (c)

Comprehension Type

10. (c) **11.** (a) **12.** (a)

Cengage Learning India Pvt. Ltd.

DPP 1.4

Composite and Inverse Functions

Single Correct Answer Type

1. If $g(x) = \left(4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7\right)^{\frac{1}{7}}$, then the value of $g(g(100))$ is equal to
(a) -1 (b) 0 (c) 1 (d) 100

2. If $f(x) = x^2 + x + \frac{3}{4}$ and $g(x) = x^2 + ax + 1$ be two real functions, then the range of a for which $g(f(x)) = 0$ has no real solution is
(a) $(-\infty, -2)$ (b) $(-2, 2)$
(c) $(-2, \infty)$ (d) $(2, \infty)$

3. If domain of $f(x)$ is $[1, 3]$, then the domain of $f(\log_2(x^2 + 3x - 2))$ is
(a) $[-5, -4] \cup [1, 2]$ (b) $[-13, -2] \cup \left[\frac{3}{5}, 5\right]$
(c) $[4, 1] \cup [2, 7]$ (d) $[-3, 2]$

4. Let $f(x) = \frac{x}{1+x}$ and let $g(x) = \frac{rx}{1-x}$. Then the number of values of r such that $f(g(x)) = g(f(x))$ for infinitely many real number x is
(a) 1 (b) 2 (c) 3 (d) 5

5. Let $f(x) = \frac{ax+b}{cx+d}$ then $f[f(x)] = x$ provided that
(a) $d = -a$ (b) $d = a$
(c) $a = b = 1$ (d) $a = b = c = d = 1$

6. If $f : R \to R$, $f(x) = x^3 + 3$, and $g : R \to R$, $g(x) = 2x + 1$, then $f^{-1}og^{-1}(23)$ equals
(a) 2 (b) 3 (c) $(14)^{1/3}$ (d) $(15)^{1/3}$

7. If $f(x) = x(x-1)$ is a function from $\left[\frac{1}{2}, \infty\right)$ to $\left[-\frac{1}{4}, \infty\right)$, then $\{x \in R : f^{-1}(x) = f(x)\}$ is
(a) null set (b) $\{0, 2\}$
(c) $\{2\}$ (d) a set containing 3 elements

8. Let $a > 1$ be a real number and $f(x) = \log_a x^2$ for $x > 0$. If f^{-1} is the inverse function of f and b and c are real numbers then $f^{-1}(b + c)$ is equal to
(a) $f^{-1}(b) \cdot f^{-1}(c)$ (b) $f^{-1}(b) + f^{-1}(c)$
(c) $\frac{1}{f(b+c)}$ (d) $\frac{1}{f^{-1}(b) + f^{-1}(c)}$

9. If the function $f(x) = \begin{cases} x+1 & \text{if } x \le 1 \\ 2x+1 & \text{if } 1 < x \le 2 \end{cases}$ and $g(x) = \begin{cases} x^2 & -1 \le x \le 2 \\ x+2 & 2 \le x \le 3 \end{cases}$ then the number of roots of the equation $f(g(x)) = 2$ is
(a) 4 (b) 3 (c) 2 (d) 1

10. Suppose $f(x) = ax + b$ and $g(x) = bx + a$, where a and b are positive integers. If $f(g(20)) - g(f(20)) = 28$, then which of the following is not true?
(a) $a = 15$ (b) $a = 6$
(c) $b = 14$ (d) $b = 3$

11. If $f(x)$ is an invertible function, and $g(x) = 2f(x) + 5$, then the value of $g^{-1}(x)$, is
(a) $2f^{-1}(x) - 5$ (b) $\frac{1}{2f^{-1}(x) + 5}$
(c) $\frac{1}{2}f^{-1}(x) + 5$ (d) $f^{-1}\left(\frac{x-5}{2}\right)$

12. If $f(x) = \begin{cases} -x+1, & x \le 0 \\ -(x-1)^2, & x \ge 1 \end{cases}$, then the number of solutions of $f(x) - f^{-1}(x) = 0$ is
(a) 0 (b) 2 (c) 3 (d) 4

Answers Key

Single Correct Answer Type

1. (d) **2.** (c) **3.** (a) **4.** (b) **5.** (a)
6. (a) **7.** (c) **8.** (a) **9.** (c) **10.** (d)
11. (d) **12.** (d)

Cengage Learning India Pvt. Ltd.

DPP 1.5

Functional Relations and Graphical Transformation

Single Correct Answer Type

1. Let $f\left(x+\frac{1}{y}\right)+f\left(x-\frac{1}{y}\right)=2f(x)f\left(\frac{1}{y}\right)\forall x,\ y\in R,\ y\neq 0$ and $f(0)=0$ then the value of $f(1)+f(2)=$

(a) -1 (b) 0 (c) 1 (d) none of these

2. If $f(x^2-6x+6)+f(x^2-4x+4)=2x\ \forall x\in R$ then $f(-3)+f(9)-5f(1)=$

(a) 7 (b) 8 (c) 9 (d) 10

3. If a function $f: R\to R$ be such that $f(x-f(y))=f(f(y))+xf(y)+f(x)-1,\ \forall x, y\in R$, then $f(2)=$

(a) 1 (b) 3 (c) -1 (d) none of these

4. If $f: R\to R$ is a function satisfying $f(x+y)=f(xy)$ for all $x, y\in R$ and $f\left(\frac{3}{4}\right)=\frac{3}{4}$, then $f\left(\frac{9}{16}\right)=$

(a) $\frac{3}{4}$ (b) $\frac{9}{16}$ (c) $\frac{\sqrt{3}}{2}$ (d) 0

5. A function $f: R\to R$ satisfies the equation $f(x)f(y)-f(xy)=x+y\ \forall x, y\in R$ and $f(1)>0$, then

(a) $f(x)f^{-1}(x)=x^2-4$ (b) $f(x)f^{-1}(x)=x^2-6$

(c) $f(x)f^{-1}(x)=x^2-1$ (d) none of these

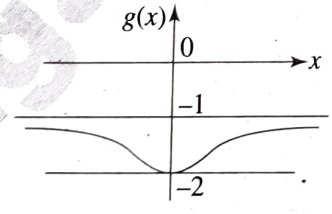
6. Let f be a function defined from $R^+\to R^+$. If $(f(xy))^2=x(f(y))^2$ for all positive numbers x and y. If $f(2)=6$, find $f(50)=?$

(a) 20 (b) 30 (c) 5 (d) 40

7. Suppose f is a real valued function satisfying $f(x+f(x))=4f(x)$ and $f(1)=4$. The value of $f(21)$ is

(a) 16 (b) 64 (c) 4 (d) 44

8. The graph of a function $y=g(x)$ is shown in the following figure. If $f(x)=-3x^2-kx-12$, $k\in R$ and $f(g(x))>0\ \forall x\in R$ then least integral value of k is equal to



(a) 13 (b) 14 (c) 15 (d) 16

9. Let $f: I\to I$ be a function (I is set of integers) such that $f(0)=1, f(f(n))=f(f(n+2)+2)=n$. Then

(a) $f(3)=0$ (b) $f(2)=0$

(c) $f(3)=-2$ (d) f is many- one function

Multiple Correct Answers Type

10. If a function satisfies $(x-y)f(x+y)-(x+y)f(x-y)=2(x^2y-y^3)\ \forall x, y\in R$ and $f(1)=2$, then

(a) $f(x)$ must be polynomial function

(b) $f(3)=12$

(c) $f(0)=0$

(d) $f(x)$ may not be differentiable

11. A function f, defined for all $x, y\in R$ is such that $f(1)=2$; $f(2)=8$ and $f(x+y)-kxy=f(x)+2y^2$, where k is some constant, then

(a) $f(0)=0$ (b) $f(0)$ cannot be determined

(c) $k=2$ (d) k cannot be determined

12. Suppose that $f: R\to R$ is continuous and satisfying the equation $f(x)\cdot f(f(x))=1$, for all real x. Let $f(1000)=999$, then which of the following is/are true?

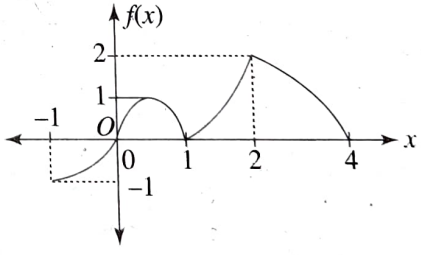
(a) $f(500)=\frac{1}{500}$

(b) $f(199)=\frac{1}{199}$

(c) $f(x)=\frac{1}{x}\ \forall\ x\in R-\{0\}$

(d) $f(1999)=\frac{1}{1999}$

13. The graph of a function $f(x)$ which is defined in $[-1, 4]$ is shown in the adjacent figure. Identify the correct statement(s).



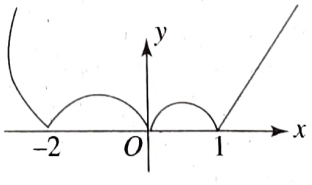
(a) domain of $f(|x|-1)$ is $[-5, 5]$

(b) range of $f(|x|+1)$ is $[0, 2]$

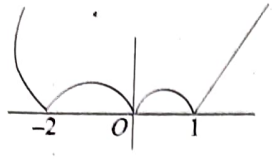
(c) range of $f(-|x|)$ is $[-1, 0]$

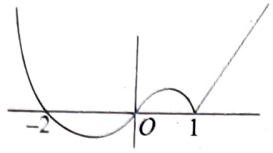
(d) domain of $f(|x|)$ is $[-3, 3]$

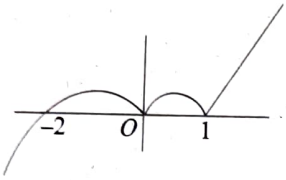
14. The graph of $y=|f(x)|$ is as shown in the following figure

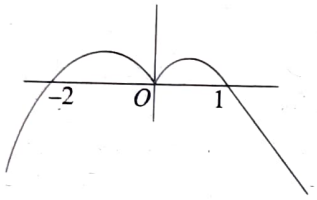


Graph of $y = f(x)$ may be

(a) 

(b) 

(c) 

(d) 

Comprehension Type

For Questions 15 and 16

Consider a differentiable function $f : R \to R$ for which $f(1) = 2$ and $f(x + y) = 2^x f(y) + 4^y f(x)\ \forall\ x, y \in R$.

15. The value of $f(4)$ is

(a) 160 (b) 240
(c) 200 (d) none of these

16. The minimum value of $f(x)$ is

(a) 1 (b) $-\frac{1}{2}$
(c) $-\frac{1}{4}$ (d) none of these

For Questions 17 and 18

Let $f(x)$ be a real valued and differentiable function on R such that $f(x + y) = \frac{f(x) + f(y)}{1 - f(x) \cdot f(y)}$

17. $f(0)$ equals

(a) 1 (b) 0 (c) -1 (d) none of these

18. $f(x)$ is

(a) odd function
(b) even function
(c) odd and even function simultaneously
(d) neither even nor odd

Answers Key

Single Correct Answer Type

1. (b) **2.** (c) **3.** (c) **4.** (a) **5.** (c)
6. (b) **7.** (b) **8.** (c) **9.** (c)

Multiple Correct Answers Type

10. (a, b, c) **11.** (a, c) **12.** (a, b)
13. (a, b, c) **14.** (a, b, c, d)

Comprehension Type

15. (b) **16.** (c) **17.** (b) **18.** (a)

Cengage Learning India Pvt. Ltd.

CHAPTER

2

Limits

DPP 2.1

Limits: Definition

Single Correct Answer Type

1. If $f: R \to R$ is defined by $f(x) = [x-3] + |x-4|$ for $x \in R$, then $\lim_{x \to 3^-} f(x)$ is equal to (where [.] represents the greatest integer function)

(a) -2 (b) -1 (c) 0 (d) 1

2. $\lim_{x \to \frac{\pi}{2}} \left[\frac{[\sin x] - [\cos x] + 1}{3}\right]$ = (where [·] denotes the greatest integer function)

(a) 0 (b) 1

(c) -1 (d) does not exist

3. $\lim_{x \to \frac{-1^-}{3}} \frac{1}{x}\left[\frac{-1}{x}\right]$ = (where [·] denotes the greatest integer function)

(a) -9 (b) -12 (c) -6 (d) 0

4. If $f(x) = \begin{cases} x + \frac{1}{2}, & x < 0 \\ 2x + \frac{3}{4}, & x \geq 0 \end{cases}$, then $\left[\lim_{x \to 0} f(x)\right]$ = (where [·] denotes the greatest integer function)

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) does not exist (d) none of these

5. $\lim_{x \to -7} \frac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)}$ = (where [.] denotes the greatest integer function)

(a) is 0 (b) is 1

(c) is -1 (d) does not exist

6. Which of the following limit exists finitely?

(a) $\lim_{x \to 0^+} (x)^{\log_e x}$

(b) $\lim_{x \to 3} \frac{x^2 - 9 - \sqrt{x^2 - 6x + 9}}{|x-1| - 2}$

(c) $\lim_{x \to 1^+} ([x])^{\frac{1}{x-1}}$ = (where [.] denotes the greatest integer function)

(d) none of these

7. Let $L_1 = \lim_{x \to 4} (x-6)^x$ and $L_2 = \lim_{x \to 4} (x-6)^4$.

Which of the following is true?

(a) Both L_1 and L_2 exists

(b) Neither L_1 nor L_2 exists

(c) L_1 exists but L_2 does not exist

(d) L_2 exists but L_1 does not exist

8. Set of all values of x such that $\lim_{n \to \infty} \frac{1}{1 + \left(\frac{4\tan^{-1}(2\pi x)}{\pi}\right)^{4n}}$ is non-zero and finite number when $n \in N$ is

Cengage Learning India Pvt. Ltd.

(a) $\left(0, \frac{1}{2\pi}\right)$ (b) $\left(-\frac{1}{\pi}, \frac{1}{\pi}\right)$

(c) $\left[-\frac{1}{2\pi}, \frac{1}{2\pi}\right]$ (d) $\left(-\frac{1}{2\pi}, 0\right)$

9. $\lim_{x\to\infty}\left[x - \log_e\left(\frac{e^x + e^{-x}}{2}\right)\right] =$

(a) $\log_e 4$ (b) 0 (c) $\log_e 2$ (d) ∞

10. $\lim_{x\to\infty}\left\{(e^x + \pi^x)^{\frac{1}{x}}\right\}$ = (where $\{.\}$ denotes the fractional part of x) is equal to

(a) $\pi - e$ (b) $\pi - 3$ (c) $e - 2$ (d) $3 - e$

11. If $\frac{\cos x}{\sin ax}$ is a periodic function, then

$\lim_{m\to\infty}\lim_{n\to\infty}(1 + \cos^{2m} n!\pi a)$ is equal to

(a) 0 (b) 1 (c) 2 (d) −1

Multiple Correct Answers Type

12. If $A = \lim_{x\to 0}\frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B = \lim_{x\to 0}\frac{[|x|]}{x}$, then

(a) $A = 1$ (b) A does not exist

(c) $B = 0$ (d) $B = 1$

13. If $f(x) = x\left(\frac{e^{|x|+[x]} - 2}{|x| + [x]}\right)$ then (where $[\cdot]$ represents the greatest integer function)

(a) $\lim_{x\to 0^+} f(x) = -1$ (b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0} f(x) = -1$ (d) $\lim_{x\to 0} f(x) = 0$

14. Assume that $\lim_{\theta\to -1} f(\theta)$ exists and $\frac{\theta^2 + \theta - 2}{\theta + 3} \le \frac{f(\theta)}{\theta^2} \le \frac{\theta^2 + 2\theta - 1}{\theta + 3}$ holds for certain interval containing the point $\theta = -1$ then $\lim_{\theta\to -1} f(\theta)$

(a) is equal to $f(-1)$ (b) is equal to 1

(c) is non-existent (d) is equal to −1

15. Let $f(x) = \lim_{n\to\infty}\frac{\tan^{-1}(\tan x)}{1 + (\log_e x)^n}$, $x \ne (2n+1)\frac{\pi}{2}$ then

(a) $\forall\, 1 < x < \frac{\pi}{2}$, $f(x)$ is an identity function

(b) $\forall\, \frac{\pi}{2} < x < \pi$, the graph of $f(x)$ is a straight line havin y intercept of $-\pi$

(c) $\forall\, \frac{\pi}{2} < x < e$, the graph of $f(x)$ is a straight line havi y intercept of $-\pi$

(d) $\forall\, x > e$, $f(x)$ is a constant function

Answers Key

Single Correct Answer Type

1. (c) **2.** (a) **3.** (c) **4.** (c) **5.** (a)
6. (c) **7.** (d) **8.** (c) **9.** (c) **10.** (b)
11. (c)

Multiple Correct Answers Type

12. (b, c) **13.** (a, b) **14.** (a, d) **15.** (a, c, d)

Cengage Learning India Pvt. Ltd.

DPP 2.2

Limits of Algebraic and Trigonometric Functions

Single Correct Answer Type

1. The value of $\lim_{x\to 0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}$ is

(a) $\frac{1}{2}$ (b) 2

(c) $\sqrt{2}$ (d) None of these

2. $\lim_{x\to\frac{\pi}{2}}(1-\sin x)\tan x =$

(a) $\frac{\pi}{2}$ (b) 1 (c) 0 (d) ∞

3. The value of $\lim_{x\to\infty} x^2\left(1-\cos\frac{1}{x}\right)$ is

(a) 0 (b) 1/4 (c) 1/2 (d) 1

4. $\lim_{x\to\infty}\sqrt[3]{x}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)=$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

5. $\lim_{n\to\infty}\frac{3\cdot 2^{n+1}-4\cdot 5^{n+1}}{5\cdot 2^n+7\cdot 5^n}=$

(a) 0 (b) 3/5 (c) –4/7 (d) –20/7

6. $\lim_{x\to 2^-}\{x\}\frac{\sin(x-2)}{(x-2)^2}=$ (where $\{.\}$ denotes the fractional part function)

(a) 0 (b) 2

(c) 1 (d) does not exist

7. $\lim_{x\to\infty}\frac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right\}}=$

(a) 1 (b) 0 (c) $\pi/2$ (d) non existent

8. $\lim_{x\to 0}\frac{3\tan 3x-4\tan 2x-\tan x}{4x^2\tan x}=$

(a) 0 (b) 1 (c) 3 (d) 4

9. $\lim_{x\to 0}\left[\frac{\sin^{-1}x}{\tan^{-1}x}\right]=$ (where [.] denotes the greatest integer function)

(a) 0 (b) 1

(c) –1 (d) none of these

10. The value of $\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi-4x}$ is

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) 1 (d) does not exist

11. The value of $\lim_{x\to\infty}\left(e^{\sqrt{x^4+1}}-e^{(x^2+1)}\right)$ is

(a) 0 (b) e (c) $1/e$ (d) $-\infty$

12. The value of $\lim_{x\to\pi/4}\frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is

(a) 8 (b) 4 (c) –8 (d) –2

13. $\lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)(8x^3-\pi^3)\cos x}{(\pi-2x)^4}$

(a) $-\frac{\pi^2}{16}$ (b) $\frac{3\pi^2}{16}$ (c) $\frac{\pi^2}{16}$ (d) $-\frac{3\pi^2}{16}$

14. $\lim_{x\to\infty}\frac{\sum_{r=1}^{10}(x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)}=$

(a) 5 (b) 2010 (c) $\frac{502}{1005}$ (d) 0

15. If $\lim_{x\to 0}\frac{f(x)}{x^2}=a$ and $\lim_{x\to 0}\frac{f(1-\cos x)}{g(x)\sin^2 x}=b$ (where $b\neq 0$), then $\lim_{x\to 0}\frac{g(1-\cos 2x)}{x^4}$ is

Cengage Learning India P

(a) $\frac{4a}{b}$ (b) $\frac{a}{4b}$

(c) $\frac{a}{b}$ (d) None of these

16. If $f(x)=\begin{cases}\frac{x}{\sin x}, & x>0\\ 2-x, & x\le 0\end{cases}$ and $g(x)=\begin{cases}x+3, & x<1\\ x^2-2x-2, & 1\le x<2\\ x-5, & x\ge 2\end{cases}$

Then the value of $\lim_{x\to 0} g(f(x))$

(a) is -2 (b) is -3

(c) is 1 (d) does not exist

17. If $k \in I$ such that $\lim_{n\to\infty}\left(\cos\frac{k\pi}{4}\right)^{2n}-\left(\cos\frac{k\pi}{6}\right)^{2n}=0$, the

(a) k must not be divisible by 24

(b) k is divisible by 24 or k is divisible neither by 4 n by 6

(c) k must be divisible by 12 but not necessarily by 24

(d) None of these

18. If a_n and b_n are positive integers and $a_n+\sqrt{2}b_n=(2+\sqrt{2})$

then $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right)=$

(a) $\sqrt{2}$ (b) 2 (c) $e^{\sqrt{2}}$ (d) e^2

Answers Key

Single Correct Answer Type

1. (c)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (d)	**11.** (d)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (c)
6. (c)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (d)	**16.** (b)	**17.** (b)	**18.** (a)		

Cengage Learning India Pvt. L

DPP 2.3

Limits of Exponential and Logarithmic Functions, Limits of Functions $f(x)^{g(x)}$ Type

Single Correct Answer Type

1. The value of $\lim_{x\to 0^+} \frac{\tan x^{\frac{1}{5}}}{(\tan^{-1}\sqrt{x})^2}\frac{\log(1+5x)}{(e^{3\sqrt[5]{x}}-1)}$ is

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c) 1 (d) none of these

2. The value of $\lim_{x\to 3}\frac{(x^3+27)\log_e(x-2)}{x^2-9}$ is

(a) 9 (b) 18 (c) 27 (d) 1/3

3. The value of $\lim_{x\to 0^+}\left(\frac{1-\cos(\sin^2 x)}{x^2}\right)^{\frac{\log_e(1-2x^2)}{\sin^2 x}}$ is

(a) 0 (b) e (c) -1 (d) ∞

4. $\lim_{x\to 0}\frac{1}{x^2}\begin{vmatrix} 1-\cos 3x & \log_e(1+4x) \\ \sin^{-1}(e^x-1) & \tan^{-1}(2x)\end{vmatrix}$ is equal to

(a) 2 (b) -4 (c) 6 (d) 4

5. If graph of the function $y = f(x)$ is continuous and passes through point (3, 1) then $\lim_{x\to 3}\frac{\log_e(3f(x)-2)}{2(1-f(x))}$ is equal

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{3}{2}$ (d) $-\frac{1}{2}$

6. Let $f(x)$ be defined for all $x \in R$ such that $\lim_{x\to 0}\left[f(x)+\log\left(1-\frac{1}{e^{f(x)}}\right)-\log(f(x))\right]=0$. Then $f(0)$ is

(a) 0 (b) 1 (c) 2 (d) 3

7. $\lim_{x\to\infty} x^2 \sin\left(\log_e\sqrt{\cos\frac{\pi}{x}}\right)$

(a) 0 (b) $-\frac{\pi^2}{2}$ (c) $-\frac{\pi^2}{4}$ (d) $-\frac{\pi^2}{8}$

8. If $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x = 4$ then the value of e^c is

(a) 1/4 (b) 1/2 (c) 1 (d) 2

9. If $\lim_{x\to 0}\left[1+x+\frac{f(x)}{x}\right]^{1/x} = e^3$, then $\lim_{x\to 0}\left[1+\frac{f(x)}{x}\right]^{1/x} =$

(a) e (b) e^2 (c) e^3 (d) none of these

10. $\lim_{x\to\frac{\pi}{2}^-}[1+(\cos x)^{\cos x}]^2 =$

(a) Does not exist (b) 1

(c) e (d) 4

11. If $a > 0, b > 0$ then $\lim_{n\to\infty}\left(\frac{a-1+b^{\frac{1}{n}}}{a}\right)^n =$

(a) $b^{\frac{1}{a}}$ (b) $a^{\frac{1}{b}}$ (c) a^b (d) b^a

12. If $f(x) = \lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$, then the value of $\lim_{x\to 0}\frac{f(x)-1}{x}$ is

(a) 0 (b) 1 (c) 2 (d) 3/2

13. $\lim_{x\to 0}\frac{\log\left(e^{x^2}+2\sqrt{x}\right)}{\tan\sqrt{x}}$ is equal to

(a) 0 (b) 1 (c) e^2 (d) 2

14. Let $f : R \to R$ be such that $f(a) = 1, f'(a) = 2$. Then $\lim_{x\to 0}\left(\frac{f^2(a+x)}{f(a)}\right)^{1/x}$ is

(a) e^2 (b) e^4 (c) e^{-4} (d) $1/e$

15. The value of $\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$ is

(a) e (b) $1/e$ (c) e^2 (d) e^{-2}

16. If $f(n) = \lim_{x\to 0}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\ldots\left(1+\sin\frac{x}{2^n}\right)\right)^{\frac{1}{x}}$ then $\lim_{n\to\infty} f(n) =$

(a) 1 (b) e (c) 0 (d) ∞

17. $\lim_{n\to\infty}(1-x+x\cdot\sqrt[n]{e})^n$ is equal to

(a) e^x (b) e^{-x}

(c) e^{2x} (d) none of these

Cengage Learning India Pvt. Ltd.

Answers Key

Single Correct Answer Type

1. (b) **2.** (a) **3.** (d) **4.** (b) **5.** (c)
6. (a) **7.** (c) **8.** (d) **9.** (b) **10.** (d)
11. (a) **12.** (a) **13.** (d) **14.** (b) **15.** (b)
16. (b) **17.** (a)

Cengage Learning India Pvt. Ltd

Limits Using L'Hospital's Rule and Expansion Formula

Single Correct Answer Type

1. The value of $\lim_{x\to 1}\frac{\sqrt[13]{x}-\sqrt[7]{x}}{\sqrt[5]{x}-\sqrt[3]{x}}$ is

(a) $\frac{44}{91}$ (b) $\frac{45}{91}$ (c) $\frac{45}{89}$ (d) $\frac{40}{93}$

2. $\lim_{x\to 1}\frac{1+\log x-x}{1-2x+x^2}=$

(a) 1 (b) -1 (c) 0 (d) $-1/2$

3. The value of $\lim_{x\to 0}\frac{1-\cos 2x}{e^{x^2}-e^x+x}$ is

(a) 1 (b) 2 (c) 4 (d) 8

4. If $f'(a)=\frac{1}{4}$, then $\lim_{h\to 0}\frac{f(a+2h^2)-f(a-2h^2)}{f(a+h^3-h^2)-f(a-h^3+h^2)}=$

(a) 0 (b) 1
(c) -2 (d) none of these

5. $\lim_{x\to 0^+}\frac{1}{x\sqrt{x}}\left(a\tan^{-1}\frac{\sqrt{x}}{a}-b\tan^{-1}\frac{\sqrt{x}}{b}\right)$ has the value equal to

(a) $\frac{a-b}{3}$ (b) 0

(c) $\frac{(a^2-b^2)}{6a^2b^2}$ (d) $\frac{a^2-b^2}{3a^2b^2}$

6. The value of $\lim_{x\to 0}\left(\frac{1+2x}{1+3x}\right)^{\frac{1}{x^2}}\cdot e^{\frac{1}{x}}$ is

(a) $e^{\frac{5}{2}}$ (b) e^2 (c) e^{-2} (d) 1

7. If $f: R\to R$ be a differentiable function at $x=0$ satisfying $f(0)=0$ and $f'(0)=1$, then the value of

$$\lim_{x\to 0}\frac{1}{x}\sum_{n=1}^{\infty}(-1)^n f\left(\frac{x}{n}\right)=$$

(a) 0 (b) $-\log 2$ (c) 1 (d) e

8. The value of $\lim_{x\to\frac{3\pi}{4}}\frac{1+\sqrt[3]{\tan x}}{1-2\cos^2 x}$ is

(a) $-1/2$ (b) $-2/3$ (c) $-3/2$ (d) $-1/3$

9. Let $g(x)=\frac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$ m and n are integers, $m\neq 0$, $n>0$ and. If $\lim_{x\to 1+} g(x)=-1$, then

(a) $n=1, m=1$ (b) $n=1, m=-1$
(c) $n=2, m=2$ (d) $n>2, m=n$

Comprehension Type

For Questions 10 and 11

Let $f(x)$ be the fourth degree polynomial such that $f'(0)=-6$, $f(0)=2$ and $\lim_{x\to 1}\frac{f(x)}{(x-1)^2}=1$

10. The value of $f(2)$ is

(a) 1 (b) 0 (c) 2 (d) 3

11. The value of $f'(2)$ is

(a) 4 (b) 5 (c) 6 (d) 7

Answers Key

Single Correct Answer Type

1. (b) **2.** (d) **3.** (c) **4.** (c) **5.** (d)
6. (a) **7.** (b) **8.** (d) **9.** (c)

Comprehension Type

10. (c) **11.** (c)

Cengage Learning India Pvt. Ltd.

DPP 2.5

Finding the Unknown

Single Correct Answer Type

1. Number of integral values of λ for which

$\lim_{x\to 1} \sec^{-1}\left(\frac{\lambda^2}{\log_e x} - \frac{\lambda^2}{x-1}\right)$ does not exist is

(a) 1 (b) 2 (c) 3 (d) 4

2. If $\lim_{x\to 0} \frac{e^{ax} - e^x - x}{x^2} = b$ (finite), then

(a) $a = 2, b = 0$ (b) $a = 0, b = \frac{3}{2}$

(c) $a = 2, b = \frac{3}{2}$ (d) $a = 0, b = 2$

3. If $\lim_{x\to 0} \frac{x^3}{\sqrt{a+x}\,(bx - \sin x)} = 1, a > 0$, then $a + b$ is equal to

(a) 36 (b) 37 (c) 38 (d) 40

4. If $\lim_{x\to\infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix}\right) = -5$, where α, β, γ are finite real numbers, then

(a) $\alpha = 2, \beta = 1, \gamma \in R$
(b) $\alpha = 2, \beta = 2, \gamma = 5$
(c) $\alpha \in R, \beta = 1, \gamma \in R$
(d) $\alpha \in R, \beta = 1, \gamma = 5$

Multiple Correct Answers Type

5. If $\lim_{x\to 0} \frac{ae^x + b\cos x + ce^{-x}}{e^{2x} - 2e^x + 1} = 4$, then

(a) $a = 2$ (b) $b = -4$
(c) $c = 2$ (d) $a + b + c = -8$

6. If $a \in I$, then value of a for which $\lim_{x\to a} \frac{\tan([x^3] - [x]^3)}{(x-a)^3}$ exists finitely, is/are

(a) 0 (b) 1 (c) −1 (d) 2

Comprehension Type

For Questions 7 and 8

$$L = \lim_{x\to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$$

7. The value/values of a is

(a) $\in R$ (b) 2 (c) 0 (d) 1

8. The value/values of b is

(a) $\in R$ (b) 2 (c) 0 (d) 1

For Questions 9 and 10

If $f(x) = \lim_{n\to\infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}}$ and $\lim_{x\to\pm 1} f(x$

exists, then

9. The value of a is

(a) −1 (b) 1 (c) 0 (d) 2

10. The value of b is

(a) −1 (b) 1 (c) 0 (d) 2

Answers Key

Single Correct Answer Type

1. (c) **2.** (c) **3.** (b) **4.** (d)

Multiple Correct Answers Type

5. (a, b, c) **6.** (a, b)

Comprehension Type

7. (a) **8.** (b) **9.** (b) **10.** (c)

Cengage Learning India Pvt. Ltd.

3 Continuity and Differentiability

DPP 3.1

Continuity (Part I)

Single Correct Answer Type

1. If $f(x)=\begin{cases}\left(\sin\dfrac{2x^2}{a}+\cos\left(\dfrac{3x}{b}\right)\right)^{ab/x^2}, & x\neq 0\\ e^3, & x=0\end{cases}$, is continuous at $x=0\ \forall\ b\in R$ then minimum value of a is

(a) $-1/8$ (b) $-1/4$
(c) $-1/2$ (d) 0

2. Let $f: R\to R$ be any function. Also, $g: R\to R$ is defined by $g(x)=|f(x)|$ for all x. Then g is

(a) onto if f is onto
(b) one-one if f is one-one
(c) continuous if f is continuous
(d) None of these

3. Let $f(x)=\begin{cases}\dfrac{1-\sin\pi x}{1+\cos 2\pi x}; & x<\dfrac{1}{2}\\ p; & x=\dfrac{1}{2}\\ \dfrac{\sqrt{2x-1}}{\sqrt{4+\sqrt{2x-1}}-2}; & x>\dfrac{1}{2}\end{cases}$. Then the value of p for which the function is continuous at $x=\dfrac{1}{2}$ is

(a) 1 (b) 1/4
(c) 4 (d) none of these

4. For which of the following functions $f(0)$ exists such that $f(x)$ is continuous at $x=0$?

(a) $f(x)=\dfrac{1}{\log_e|x|}$ (b) $f(x)=\cos\left(\dfrac{|\sin x|}{x}\right)$

(c) $f(x)=x\sin\dfrac{\pi}{x}$ (d) $f(x)=\dfrac{1}{1+2^{\cot x}}$

5. Let $f(x)=x^3-x^2-3x-1$, $g(x)=(x+1)a$ and $h(x)=\dfrac{f(x)}{g(x)}$ where h is a rational function such that

(i) it is continuous everywhere except when $x=-1$.
(ii) $\lim\limits_{x\to\infty}h(x)=\infty$ and
(iii) $\lim\limits_{x\to -1}h(x)=\dfrac{1}{2}$.

The value of $h(1)$ is

(a) 1/2 (b) 1/4 (c) $-1/2$ (d) 1

6. If the function $f(x)=\dfrac{3x^2+ax+a+3}{x^2+x-2}$ is continuous at $x=-2$, then the value of $f(-2)$ is

(a) 0 (b) -1 (c) 1 (d) 2

7. Let $f(x)=\begin{cases}8^{\frac{1}{x}}, & x<0\\ a[x], & a\in R-\{0\},\ x\geq 0,\end{cases}$ (where [.] denotes the greatest integer function).
Then $f(x)$ is

(a) continuous only at a finite number of points.
(b) discontinuous at a finite number of points.

Cengage Learning India Pvt. Ltd.

(c) discontinuous at an infinite number of points.
(d) discontinuous at $x = 0$.

8. If $f(x) = \begin{cases} \dfrac{2\cos x - \sin 2x}{(\pi - 2x)^2}, & x \le \dfrac{\pi}{2} \\ \dfrac{e^{-\cos x} - 1}{8x - 4\pi}, & x > \dfrac{\pi}{2} \end{cases}$ then which of the following holds?

(a) f is continuous at $x = \pi/2$
(b) f has an irremovable discontinuity at $x = \pi/2$
(c) f has a removable discontinuity at $x = \pi/2$
(d) None of these

9. If $f(x) = \begin{cases} \sin\left(\dfrac{\pi}{2}(x - [x])\right), & x < 5 \\ 5(b-1), & x = 5 \\ ab^2 \dfrac{|x^2 - 11x + 24|}{x - 3}, & x > 5 \end{cases}$ is continuous at $x = 5$, $a, b \in R$ then ([·] denotes the greatest integer function)

(a) $a = \dfrac{25}{108}, b = \dfrac{6}{5}$ (b) $a = \dfrac{6}{13}, b = \dfrac{17}{29}$

(c) $a = \dfrac{1}{2}, b = \dfrac{25}{36}$ (d) $a = \dfrac{23}{100}, b = \dfrac{6}{5}$

10. The function $f(x)$ is discontinuous only at $x = 0$ such that $f^2(x) = 1\ \forall\ x \in R$. The total number of such functions is
(a) 2 (b) 3
(c) 6 (d) None of these

11. $f(x) = \begin{cases} (x^2 + e^{\frac{1}{2-x}})^{-1}, & x \ne 2 \\ k, & x = 2 \end{cases}$ is continuous from right at the point $x = 2$, then k equals
(a) 0 (b) 1/4
(c) −1/4 (d) none of these

12. The number of points of discontinuity of $g(x) = f(f(x))$ where $f(x)$ is defined as, $f(x) = \begin{cases} 1 + x, & 0 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \end{cases}$
(a) 0 (b) 1 (c) 2 (d) > 2

13. If the function $f(x) = \dfrac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$ is continuous at $x = 0$, then the value of $\dfrac{a}{b}$ is
(a) $\dfrac{3}{5} f(0)$ (b) $2^{8/5} f(0)$
(c) $\dfrac{64}{5} f(0)$ (d) none of these

14. If $f(x) = \begin{cases} \dfrac{1 - \cos\left(1 - \cos\dfrac{x}{2}\right)}{2^m x^n}, & x \ne 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$ then the value of $m + n$ is
(a) 2 (b) 3 (c) −3 (d) 7

Multiple Correct Answers Type

15. Which of the following functions is/are discontinuous at $x = 1$?
(a) $f(x) = \dfrac{1}{1 + 2^{\tan x}}$
(b) $g(x) = \lim\limits_{n \to \infty} \dfrac{1}{1 + n\sin^2(\pi x)}$
(c) $h(x) = 2^{-2^{\left(\frac{1}{1-x}\right)}}, x \ne 1$ and $h(1) = 1$
(d) $\phi(x) = \dfrac{x - 1}{|x - 1| + 2(x - 1)^2}, x \ne 1$ and $\phi(1) = 1$

16. f is a continous function in $[a, b]$ and g is a continuous functin in $[b, c]$. A function $h(x)$ is defined as

$$h(x) = \begin{cases} f(x), & x \in [a, b) \\ g(x), & x \in (b, c] \end{cases}$$

Now, if $f(b) = g(b)$, then
(a) $h(x)$ may or may not be continuous in $[a, c]$
(b) $h(b^+) = g(b^-)$ and $h(b^-) = f(b^+)$
(c) $h(b^-) = g(b^+)$ and $h(b^+) = f(b^-)$
(d) $h(x)$ has a removable discontinuity at $x = b$

17. If the function $f(x)$ defined as $f(x) = \begin{cases} 3, & x = 0 \\ \left(1 + \dfrac{ax + bx^3}{x^2}\right), & x > 0 \end{cases}$ is continuous at $x = 0$, then
(a) $a = 0$ (b) $b = e^3$
(c) $a = 1$ (d) $b = \log_e 3$

Answers Key

Single Correct Answer Type

1. (b)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (c)
6. (b)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	

Multiple Correct Answers Type

15. (b, c, d) **16.** (c, d) **17.** (a, d)

Cengage Learning India Pvt. Ltd.

DPP 3.2

Continuity (Part II)

Single Correct Answer Type

1. Let $f(x) = \begin{cases} \frac{\alpha \cot x}{x} + \frac{\beta}{x^2}, & 0 < |x| \le 1 \\ \frac{1}{3}, & x = 0 \end{cases}$

If $f(x)$ is continuous at $x = 0$, then the value of $\alpha^2 + \beta^2$ is
(a) 1 (b) 2 (c) 5 (c) 9

2. If $f(x) = \begin{cases} \frac{2}{1+x^2}, & x \text{ is irrational} \\ b, & x \text{ is rational} \end{cases}$ has exactly two points of continuity then the values of b are
(a) (0, 3] (b) [0, 1]
(c) (0, 2] (d) ϕ

3. If $f(x) = \begin{cases} \sin\left(\frac{a-x}{2}\right)\tan\left[\frac{\pi x}{2a}\right] & \text{for } x > a \\ \frac{\left[\cos\left(\frac{\pi x}{2a}\right)\right]}{a-x} & \text{for } x < a \end{cases}$

(where $[x]$ is the greatest integer function of x), and $a > 0$, then
(a) $f(a^-) < 0$
(b) f has a removable discontinuity at $x = a$
(c) f has an irremovable discontinuity at $x = a$
(d) $f(a^+) < 0$

4. Let $f(x) = [\tan x [\cot x]]$, $x \in \left[\frac{\pi}{12}, \frac{\pi}{2}\right)$, (where $[\cdot]$ denotes the greatest integer less than or equal to x). Then the number of points, where $f(x)$ is discontinuous is
(a) one (b) zero
(c) three (d) infinite

5. Let $f : [a, b] \to R$ be any function which is such that $f(x)$ is rational for irrational x and $f(x)$ is irrational for rational x. Then in $[a, b]$
(a) f is discontinuous everywhere
(b) f is continuous only at $x = 0$ and discontinuous everywhere
(c) f is continuous for all irrational x and discontinuous for rational x
(d) f is continuous for rational x and discontinuous for irrational x

6. If $f(x) = [x] (\sin kx)^p$ is continuous for real x, then (where $[\cdot]$ represents the greatest integer function)
(a) $k \in \{n\pi, n \in I\}, p > 0$
(b) $k \in \{2n\pi, n \in I\}, p > 0$
(c) $k \in \{n\pi, n \in I\}, p \in R - \{0\}$
(d) $k \in \{n\pi, n \in I, n \ne 0\}, p \in R - \{0\}$

7. **Statement 1:** Minimum number of points of discontinuity of the function $f(x) = (g(x))\,[2x - 1]\ \forall\ x \in (-3, -1)$, where [.] denotes the greatest integer function and $g(x) = ax^3 + x^2 + 1$ is zero.
Statement 2: $f(x)$ can be continuous at a point of discontinuity, say $x = c_1$ of $[2x - 1]$ if $g(c_1) = 0$.
(a) Statement 1 is True, Statement 2 is True, Statement 2 is a correct explaination for Statement 1.
(b) Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for Statement 1.
(c) Statement 1 is True, Statement 2 is False.
(d) Statement 1 is False, Statement 2 is True.

8. Number of points of discontinuity of $f(x) = [\sin^{-1} x] - [x]$ in its domain is equal to (where $[\cdot]$ denotes the greatest integer function)
(a) 0 (b) 1 (c) 2 (d) 3

9. If $g(x) = \lim_{m \to \infty} \frac{x^m f(x) + h(x) + 3}{2x^m + 4x + 1}$ when $x \ne 1$ and $g(1) = e^3$ such that $f(x)$, $g(x)$ and $h(x)$ are continuous functions at $x = 1$ then the value of $5f(1) - 2h(1)$ is
(a) 7 (b) 6 (c) 9 (d) 8

10. The number of points of discontinuity of $f(x) = [2x]^2 - \{2x\}^2$ (where [ ] denotes the greatest integer function and { } is fractional part of x) in the interval (–2, 2), is
(a) 1 (b) 6 (c) 2 (d) 4

11. Let $f(x) = \begin{cases} -3 + |x|, & -\infty < x < 1 \\ a + |2 - x|, & 1 \le x < \infty \end{cases}$ and

$g(x) = \begin{cases} 2 - |-x|, & -\infty < x < 2 \\ -b + \text{sgn}(x), & 2 \le x < \infty \end{cases}$

where sgn(x) denotes signum function of x. If $h(x) = f(x) + g(x)$ is discontinuous at exactly one point, then which of the following is not possible?
(a) $a = -3, b = 0$ (b) $a = 0, b = 1$
(c) $a = 2, b = 1$ (d) $a = -3, b = 1$

College Learning Pvt. Ltd.

12. The function $f(x) = \frac{x^3}{8} - \sin \pi x + 4$ in $[-4, 4]$ does not take the value
(a) -4 (b) 10 (c) 18 (d) 12

13. Let $f(x)$ be continuous functions $f : R \to R$ satisfying $f(0) = 1$ and $f(2x) - f(x) = x$. Then the value of $f(3)$ is
(a) 2 (b) 3 (c) 4 (d) 5

Multiple Correct Answers Type

14. Given $f(x) = \begin{cases} \dot{3} - \left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right] & \text{for } x > 0 \\ \{x^2\}\cos(e^{1/x}) & \text{for } x < 0 \end{cases}$ (where $\{\}$ and $[]$ denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does not hold good?
(a) $f(0^-) = 0$
(b) $f(0^+) = 3$
(c) If $f(0) = 0$, then $f(x)$ is continuous at $x = 0$
(d) Irremovable discontinuity of f at $x = 0$

15. Let $f(x) = \begin{cases} x\left[\frac{1}{x}\right] + x[x] & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (where $[x]$ denotes the greatest integer function). Then the correct statements is/are
(a) Limit exists for $x = -1$.
(b) $f(x)$ has a removable discontinuity at $x = 1$.
(c) $f(x)$ has a non removable discontinuity at $x = 2$.
(d) $f(x)$ is discontinuous at all positive integers.

16. A function $f : R \to R$ is defined as
$f(x) = \lim_{n\to\infty} \frac{ax^2 + bx + c + e^{nx}}{1 + c \cdot e^{nx}}$ where f is continuous on R, then
(a) point (a, b, c) lies on line in space
(b) point (a, b) represents the 2-dimensional Cartesian plane
(c) Locus of points (a, c) and (c, b) intersect at one point
(d) point (a, b, c) lies on the plane in space

Answers Key

Single Correct Answer Type

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)
6. (a)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (c)		

Multiple Correct Answers Type

14. (b, d) **15.** (a, b, c, d) **16.** (a, b, c)

Cengage Learning India Pvt. Ltd.

DPP 3.3

Differentiability

Single Correct Answer Type

1. Let $f(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & \text{for } x < 1 \\ ax + b & \text{for } x \geq 1 \end{cases}$.

If $f(x)$ is the continuous and differentiable for all real values in its domain then

(a) $a = b = 4$ (b) $a = b = -4$

(c) $a = 4$ and $b = -4$ (d) $a = -4$ and $b = 4$

2. If $f(x) = \begin{cases} [x] + \sqrt{\{x\}}, & x < 1 \\ \dfrac{1}{[x] + \{x\}^2}, & x \geq 1 \end{cases}$, then

[where $[\cdot]$ and $\{\cdot\}$ represent the greatest integer and fractional part functions respectively]

(a) $f(x)$ is continuous at $x = 1$ but not differentiable

(b) $f(x)$ is not continuous at $x = 1$

(c) $f(x)$ is differentiable at $x = 1$

(d) $\lim\limits_{x \to 1} f(x)$ does not exist

3. If f is an even function such that $\lim\limits_{h \to 0} \dfrac{f(h) - f(0)}{h}$ has some finite non-zero value, then

(a) f is continuous and derivable at $x = 0$

(b) f is continuous but not differentiable at $x = 0$

(c) f may be discontinuous at $x = 0$

(d) None of these

4. Let $f(x)$ be differentiable for real x such that

$f'(x) > 0$ on $(-\infty, -4)$,

$f'(x) < 0$ on $(-4, 6)$ and

$f'(x) > 0$ on $(6, \infty)$.

If $g(x) = f(10 - 2x)$, then the value of $g'(2)$ is

(a) 1 (b) 2 (c) 0 (d) 4

5. Number of points where $f(x) = x^2 - |x^2 - 1| + 2||x| - 1| + 2|x| - 7$ is non-differentiable is

(a) 0 (b) 1 (c) 2 (d) 3

6. If $f(x) = |x - 1| \cdot ([x] - [-x])$, then (where $[\cdot]$ represents greatest integer function)

(a) $f(x)$ is continuous and differentiable at $x = 1$

(b) $f(x)$ is discontinuous at $x = 1$

(c) $f(x)$ is continuous at $x = 2$

(d) $f(x)$ is continuous but non-differentiable at $x = 1$

7. Number of points where function $f(x)$ defined as $f : [0, 2\pi] \to R, f(x) = \begin{cases} 3 - \left|\cos x - \dfrac{1}{\sqrt{2}}\right|, & |\sin x| < \dfrac{1}{\sqrt{2}} \\ 2 + \left|\cos x + \dfrac{1}{\sqrt{2}}\right|, & |\sin x| \geq \dfrac{1}{\sqrt{2}} \end{cases}$ is non-differentiable is

(a) 2 (b) 4 (c) 6 (d) 0

8. Let $f(x) = \begin{cases} [x] & x \notin I \\ x - 1 & x \in I \end{cases}$ (where, $[\cdot]$ denotes the greatest integer function) and

$g(x) = \begin{cases} \sin x + \cos x, & x < 0 \\ 1, & x \geq 0 \end{cases}$. Then for $f(g(x))$ at $x = 0$

(a) $\lim\limits_{x \to 0} f(g(x))$ exists but not continuous

(b) continuous but not differentiable at $x = 0$

(c) differentiable at $x = 0$

(d) $\lim\limits_{x \to 0} f(g(x))$ does not exist

9. If $f(x) = \begin{cases} \sin(\cos^{-1} x) + \cos(\sin^{-1} x), & x \leq 0 \\ \sin(\cos^{-1} x) - \cos(\sin^{-1} x), & x > 0 \end{cases}$ then at $x = 0$

(a) $f(x)$ is continuous and differentiable

(b) $f(x)$ is continuous but not differentiable

(c) $f(x)$ not continuous but differentiable

(d) $f(x)$ is neither continuous nor differentiable

10. If $f(x) = \max\{\tan x, \sin x, \cos x\}$ where $x \in \left[-\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$ then the number of points, where $f(x)$ is non-differentiable, is

(a) 2 (b) 3 (c) 4 (d) 5

11. The number of points at which $g(x) = \dfrac{1}{1 + \dfrac{2}{f(x)}}$ is not differentiable, where $f(x) = \dfrac{1}{1 + \dfrac{1}{x}}$, is

(a) 1 (b) 2 (c) 3 (d) 4

Career Learning India Pvt. Ltd.

12. Let $f(x)=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{x}{(rx+1)\{(r+1)x+1\}}$. Then

(a) $f(x)$ is continuous but not differentiable at $x=0$
(b) $f(x)$ is both continuous and differentiable at $x=0$
(c) $f(x)$ is neither continuous not differentiable at $x=0$
(d) $f(x)$ is a periodic function

13. Let the given function is differentiable at $x=1$.

$$f(x)=\begin{cases}\lim_{n\to\infty}\dfrac{ax(x-1)\left(\cot\frac{\pi x}{4}\right)^n+(px^2+2)}{\left(\cot\frac{\pi x}{4}\right)^n+1}, & x\in(0,1)\cup(1,2)\\ 0, & x=1\end{cases}$$

Then the value of $|a+p|$ is

(a) 4 (b) 6 (c) 8 (d) 10

Multiple Correct Answers Type

14. Let f be a function with continuous second derivative and $f(0)=0, f'(0)=0$. Define a function g by

$$g(x)=\begin{cases}\frac{f(x)}{x}, & x\neq 0\\ 0, & x=0\end{cases}$$

Then which of the following statements is correct?

(a) g has a continuous first derivative
(b) g has a first derivative
(c) g is continuous but g fails to have a derivative
(d) g has a first derivative but the first derivative is not continuous

15. Let $f(x)$ be a function defined on $(-a, a)$ with $a>0$. Assume that $f(x)$ is continuous at $x=0$ and $\lim_{x\to 0}\frac{f(x)-f(kx)}{x}=\alpha$, where $k\in(0,1)$ then

(a) $f'(0^+)=0$
(b) $f'(0^-)=\dfrac{\alpha}{1-k}$
(c) $f(x)$ is differentiable at $x=0$
(d) $f(x)$ is non-differentiable at $x=0$

16. If $f(x)=x^{1/3}(x-2)^{2/3}$ for all x, then the domain of f' is

(a) $x\in R-\{0\}$ (b) $\{x\mid x>0\}$
(c) $x\in R-\{0,2\}$ (D) $x\in R$

Comprehension Type

For Questions 17 and 18

Consider two functions $y=f(x)$ and $y=g(x)$ defined as

$$f(x)=\begin{cases}ax^2+b; & 0\le x\le 1\\ bx+2b; & 1<x\le 3\\ (a-1)x+2c-3; & 3<x\le 4\end{cases}$$

and $$g(x)=\begin{cases}cx+d; & 0\le x\le 2\\ ax+3-c; & 2<x<3\\ x^2+b+1; & 3\le x\le 4\end{cases}$$

17. Let f be differentiable at $x=1$ and $g(x)$ be continuous at $x=3$. If the roots of the quadratic equation $x^2+(a+b+c)\alpha x+49(k+k\alpha)=0$ are real distinct for all values of α then possible values of k will be

(a) $k\in(-1,0)$ (b) $k\in(\infty,0)$
(c) $k\in(1,5)$ (d) $k\in(-1,1)$

18. $\lim_{x\to 2}\frac{f(x)}{|g(x)|+1}$ exists and f is differentiable at $x=1$. The value of limit will be

(a) -2 (b) -1 (c) 0 (d) 2

Answers Key

Single Correct Answer Type

1. (c) **2.** (a) **3.** (b) **4.** (c) **5.** (a)
6. (d) **7.** (b) **8.** (c) **9.** (d) **10.** (b)
11. (c) **12.** (c) **13.** (b)

Multiple Correct Answers Type

14. (a, b) **15.** (b, c) **16.** (c)

Comprehension Type

17. (a) **18.** (c)

Cengage Learning India Pvt. Ltd.

4 Methods of Differentiation

DPP 4.1

Differentiation: Definition, Different Rules of Differentiation

Single Correct Answer Type

1. The right hand derivative of $f(x) = [x] \tan \pi x$ at $x = 7$ is (where [.] denotes the greatest integer function)
(a) 0 (b) 7π
(c) -7π (d) none of these

2. Let $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and $g(x - y) = g(x) \cdot g(y) - f(x) \cdot g(y)$ for all $x, y \in R$. If right hand derivative at $x = 0$ exists for $f(x)$, then derivative of $g(x)$ at $x = 0$
(a) -1 (b) 0
(c) 1 (d) none of these

3. If $xe^{xy} - y = \sin^2 x$ then $\frac{dy}{dx}$ at $x = 0$ is
(a) 0 (b) 1
(c) -1 (d) None of these

4. Let f, g and h be differentiable functions. If $f(0) = 1$; $g(0) = 2$; $h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are $(fg)'(0) = 6$; $(gh)'(0) = 4$ and $(hf)'(0) = 5$ then the value of $((fgh)'(0))/2$ is
(a) 2 (b) 4 (c) 8 (d) 16

5. If for a continuous function f, $f(0) = f(1) = 0$, $f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to
(a) 1 (b) 2
(c) 0 (d) none of these

6. The derivative of $\cos\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) - 2\cos^{-1}\sqrt{\frac{1-x}{2}}$ w. r. t. x is
(a) $1 - \frac{1}{\sqrt{1-x^2}}$ (b) $1 - \frac{1}{\sqrt{1+x^2}}$
(c) $2 - \frac{1}{\sqrt{1-x^2}}$ (d) $2 - \frac{1}{\sqrt{1+x^2}}$

7. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2+1} + \ln\sqrt{x+\sqrt{x^2+1}}$ then the value of $xy' + \log y'$ is
(a) y (b) $2y$ (c) 0 (d) $-2y$

8. Let $g(x) = f(x) \sin x$, where $f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. Then value of $g''(-\pi)$ equals
(a) 1 (b) 2 (c) -2 (d) 0

9. If $f(x) = \log_x(\log_e x)$ then $f'(x)$ at $x = e$ is
(a) 0 (b) 1 (c) e (d) $1/e$

10. Let $g(x) = e^{f(x)}$ and $f(x + 1) = x + f(x)$ $\forall x \in R$. If $n \in I^+$, then $\frac{g'\left(n+\frac{1}{2}\right)}{g\left(n+\frac{1}{2}\right)} - \frac{g'\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)} =$
(a) $2\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$
(b) $2\left(1 + \frac{1}{3} + \frac{1}{5} + \cdots \frac{1}{2n-1}\right)$
(c) n
(d) 1

College Learning India Pvt. Ltd.

11. $\frac{d}{dx}[\cos^{-1}(x\sqrt{x}-\sqrt{(1-x)(1-x^2)})]=$

(a) $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

(b) $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

(c) $\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$

(d) $\frac{1}{\sqrt{1-x^2}}$

Multiple Correct Answers Type

12. $y=\cos^{-1}\sqrt{\frac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2(1+x^2)}, x\in R$ (b) $\frac{1}{2(1+x^2)}, x>0$

(c) $\frac{1}{2(1+x^2)}, x<0$ (d) $\frac{-1}{2(1+x^2)}, x<0$

13. Suppose that $f(x)$ is a differentiable invertible functio $f'(x)\neq 0$ and $h'(x)=f(x)$. Given that $f(1)=f'(1)=1$ $h(1)=0$ and $g(x)$ is inverse of $f(x)$. Let $G(x)=x^2g(x)-xh(g(x)$ $\forall\, x\in R$. Which of the following is/are correct?

(a) $G'(1)=2$ (b) $G'(1)=3$

(c) $G''(1)=2$ (d) $G''(1)=3$

Answers Key

Single Correct Answer Type

1. (b)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (b)
6. (a)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (c)
11. (b)				

Multiple Correct Answers Type

12. (b, d) **13.** (a, d)

Cengage Learning India Pvt. Ltd.

DPP 4.2

Differentiation of Function in Parametric Form, Differentiation of One Function w.r.t. Another Function, Differentiation Using Logarithm

Single Correct Answer Type

1. If $t(1+x^2) = x$ and $x^2 + t^2 = y$ then at $x = 2$, the value of $\frac{dy}{dx}$ is

(a) $\frac{24}{5}$ (b) $\frac{101}{125}$ (c) $\frac{488}{125}$ (d) $\frac{358}{125}$

2. If $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$ satisfy $f(x)\left(\frac{dy}{dx}\right)^3 = 1 + \frac{dy}{dx}$, then $f(x)$ equals

(a) x (b) $\frac{x^2}{1+x^2}$

(c) $x + x + \frac{1}{x}$ (d) $x - \frac{1}{x}$

3. Let $y = x^3 - 8x + 7$ and $x = f(t)$. If $\frac{dy}{dt} = 2$ and $x = 3$ at $t = 0$, then $\frac{dx}{dt}$ at $t = 0$ is given by

(a) 1 (b) $\frac{19}{2}$

(c) $\frac{2}{19}$ (d) none of these

4. If $x + \cos\theta = \sec\theta$, $y + \cos^8\theta = \sec^8\theta$ then $\left(\frac{x^2+4}{y^2+4}\right)\left(\frac{dy}{dx}\right)^2 =$

(a) 8 (b) 16 (c) 64 (d) 49

5. The derivative of the function represented parametrically as $x = 2t - |t|$, $y = t^3 + t^2|t|$ at $t = 0$ is

(a) -1 (b) 0

(c) 1 (d) does not exist

6. If $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ and $x = \sec^{-1}\frac{1}{2u^2-1}$, $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$ then the value of $2\frac{dy}{dx} + 1$ is

(a) y (b) xy (c) 0 (d) 1

7. The differential coefficient of $\sin^{-1}\left(\frac{4\sin 2x + 3\cos 2x}{5}\right)$ with respect to $\cos^{-1}\left(\frac{5\cos x - 4\sin x}{\sqrt{41}}\right)$ is

(a) -2 (b) -1 (c) 1 (d) 2

8. $xy = (x+y)^n$ and $\frac{dy}{dx} = \frac{y}{x}$ then $n =$

(a) 1 (b) 2 (c) 3 (d) 4

9. If $x + y = 3e^2$ then $\frac{d}{dx}(x^y) = 0$ for $x =$

(a) e (b) e^2 (c) e^e (d) $2e^2$

10. If $f(x) = (x-1)^{100}(x-2)^{2(99)}(x-3)^{3(98)} \ldots (x-100)^{100}$, then the value of $\frac{f'(101)}{f(101)}$ is

(a) 5050 (b) 2575 (c) 3030 (d) 1250

Multiple Correct Answers Type

11. A curve in the xy-plane is parametrically given by $x = t + t^3$ and $y = t^2$, where $t \in R$ is the parameter. For what value(s) of t is $\frac{dy}{dx} = \frac{1}{2}$?

(a) $\frac{1}{3}$ (b) 2 (c) 3 (d) 1

12. If $y = x^{(\log x)^{\log(\log x)}}$, then $\frac{dy}{dx}$ is

(a) $\frac{y}{x}(\ln x^{\log x - 1}) + 2\ln x \ln(\ln x))$

(b) $\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x) + 1)$

(c) $[(\ln x)^2 + 2\ln(\ln x)]$

(d) $\frac{y}{x}\frac{\log y}{\log x}(2\log(\log x) + 1)$

Answers Key

Single Correct Answer Type

1. (c) **2.** (a) **3.** (c) **4.** (c) **5.** (b)
6. (c) **7.** (d) **8.** (b) **9.** (b) **10.** (a)

Multiple Correct Answers Type

11. (a, d) **12.** (b, d)

Cengage Learning India Pvt.

DPP 4.3

Higher Order Derivatives

Single Correct Answer Type

1. The function $f: R \to R$ satisfies $f(x^2) \cdot f''(x) = f'(x) \cdot f'(x^2)$ for all real x. Given that $f(1) = 1$ and $f'''(1) = 8$, then the value of $f'(1) + f''(1)$ is
(a) 2 (b) 4 (c) 6 (d) 8

2. The second derivative of a single valued function parametrically represented by $x = \phi(t)$ and $y = \Psi(t)$ (where $\phi(t)$ and $\Psi(t)$ are different functions and $\phi'(t) \neq 0$) is given by

(a) $\dfrac{d^2y}{dx^2} = \dfrac{\left(\dfrac{dx}{dt}\right)\left(\dfrac{d^2y}{dt^2}\right) - \left(\dfrac{d^2x}{dt^2}\right)\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)^3}$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\left(\dfrac{dx}{dt}\right)\left(\dfrac{d^2y}{dt^2}\right) - \left(\dfrac{d^2x}{dt^2}\right)\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)^2}$

(c) $\dfrac{d^2y}{dx^2} = \dfrac{\left(\dfrac{d^2x}{dt}\right)\left(\dfrac{dy}{dt}\right) - \dfrac{dx}{dt}\left(\dfrac{d^2y}{dt^2}\right)}{\left(\dfrac{dx}{dt}\right)^3}$

(d) $\dfrac{d^2y}{dx^2} = \dfrac{\left(\dfrac{d^2x}{dt^2}\right)\left(\dfrac{dy}{dt}\right) - \left(\dfrac{d^2y}{dt^2}\right)\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dy}{dt}\right)^3}$

3. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim\limits_{x \to 0} x^{\alpha} \dfrac{d^2y}{dx^2}$ exists and non-zero then $2\alpha =$
(a) 3 (b) 4 (c) 5 (d) 1

4. If $y = \left(\dfrac{\alpha x + \beta}{\gamma x + \delta}\right)$, then $2\dfrac{dy}{dx} \cdot \dfrac{d^3y}{dx^3}$ is
(a) $7\left(\dfrac{d^2y}{dx^2}\right)^2$
(b) $5\left(\dfrac{d^2y}{dx^2}\right)^2$
(c) $3\left(\dfrac{d^2y}{dx^2}\right)^2$
(d) $\left(\dfrac{d^2y}{dx^2}\right)^2$

5. If $f(1) = 3, f'(1) = 2, f''(1) = 4$, then $(f^{-1})''(3) =$
(a) 1 (b) $-\dfrac{1}{2}$ (c) -2 (d) none of these

6. If the third derivative of $\dfrac{x^4}{(x-1)(x-2)}$ is $\dfrac{-12k}{(x-2)^4} + \dfrac{6}{(x-1)^4}$, then the value of k is
(a) 2 (b) 4 (c) 8 (d) 16

7. If $(a + bx)\, e^{y/x} = x$ then $x^3 \dfrac{d^2y}{dx^2}$ equals
(a) $\left(\dfrac{dy}{dx} + x\right)^2$ (b) $\left(x\dfrac{dy}{dx} - y\right)^2$
(c) $\left(\dfrac{dy}{dx} - y\right)^2$ (d) $\left(x\dfrac{dy}{dx} + y\right)^2$

8. If $R = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$, then $R^{2/3}$ can be put in the form of
(a) $\dfrac{1}{\left(\dfrac{d^2y}{dx^2}\right)^{2/3}} + \dfrac{1}{\left(\dfrac{d^2x}{dy^2}\right)^{2/3}}$ (b) $\dfrac{1}{\left(\dfrac{d^2y}{dx^2}\right)^{2/3}} + \dfrac{1}{\left(\dfrac{d^2x}{dy^2}\right)^{2/3}}$
(c) $\dfrac{2}{\left(\dfrac{d^2y}{dx^2}\right)^{2/3}} + \dfrac{2}{\left(\dfrac{d^2x}{dy^2}\right)^{2/3}}$ (d) $\dfrac{1}{\left(\dfrac{d^2y}{dx^2}\right)^{2/3}} \cdot \dfrac{1}{\left(\dfrac{d^2x}{dy^2}\right)^{2/3}}$

Cengage Learning India Pvt. Ltd.

9. If $x = 2\cos t - \cos 2t$ and $y = 2\sin t - \sin 2t$, then the value of (d^2y/dx^2) when $t = \pi/2$.
(a) 1/2 (b) 5/2 (c) −3/2 (d) 2

10. If $y^3 - y = 2x$, then $\left(x^2 - \frac{1}{27}\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$
(a) y (b) $\frac{y}{3}$ (c) $\frac{y}{9}$ (d) $\frac{y}{27}$

Multiple Correct Answers Type

11. If $y = e^{-x}\cos x$ and $y_n + k_n y = 0$ where $y_n = \frac{d^n y}{dx^n}$ and k_n, are constants $\forall\ n \in N$, then
(a) $k_4 = 4$ (b) $k_8 = -16$
(c) $k_{12} = 20$ (d) $k_{16} = -24$

12. If $y = y(x)$ and it follows the relation $e^{xy} + y\cos x = 2$, then
(a) $y'(0) = -1$ (b) $y''(0) = 2$
(c) $y'(0) = 3/2$ (d) $y''(0) = -2$

13. A twice differentiable function $f(x)$ is defined for all real numbers and satisfies the following conditions: $f(0) = 2$; $f'(0) = -5$ and $f''(0) = 3$. The function $g(x)$ is defined by $g(x) = e^{ax} + f(x)\ \forall\ x \in R$, where '$a$' is any constant. If $g'(0) + g''(0) = 0$. Then the value/values of a is/are
(a) 1 (b) −1 (c) 2 (d) −2

Answers Key

Single Correct Answer Type

1. (c) **2.** (a) **3.** (a) **4.** (c) **5.** (b)
6. (c) **7.** (b) **8.** (a) **9.** (c) **10.** (c)

Multiple Correct Answers Type

11. (a, b) **12.** (a, b) **13.** (a, d)

Cengage Learning India Pvt. Ltd.

DPP 4.4

Differentiation Using First Definition of Derivatives and Miscellaneous Problems

Single Correct Answer Type

1. Let $f(x) = \frac{g(x)}{x}$ when $x \neq 0$ and $f(0) = 0$.

   If $g(0) = g'(0) = 0$ and $g''(0) = 17$ then $f'(0) =$

   (a) 3/4 (b) −1/2 (c) 17/3 (d) 17/2

2. Let $f: (-\infty, \infty) \to [0, \infty)$ be a continuous function such that $f(x+y) = f(x) + f(y) + f(x)f(y)$, $\forall x, y \in R$. Also $f'(0) = 1$. Then $[f(2)]$ equals ($[\cdot]$ represents the greatest integer function)

   (a) 5 (b) 6 (c) 7 (d) 8

3. Let $f: R \to R$ be a function satisfying $f(x+y) = f(x) + \lambda xy + 3x^2y^2$ for all $x, y \in R$. If $f(3) = 4$ and $f(5) = 52$ then $f'(x)$ is equal to

   (a) $10x$ (b) $-10x$ (c) $20x$ (d) $128x$

4. A function $f: R \to [1, \infty)$ satisfies the equation $f(xy) = f(x)f(y) - f(x) - f(y) + 2$. If f is differentiable on $R - \{0\}$ and $f(2) = 5$, $f'(x) = \frac{f(x)-1}{x} \cdot \lambda$ then $\lambda =$

   (a) $2f'(1)$ (b) $3f'(1)$ (c) $\frac{1}{2}f'(1)$ (d) $f'(1)$

5. Let $\frac{f(x+y)-f(x)}{2} = \frac{f(y)-a}{2} + xy$ for all real x and y. If $f(x)$ is differentiable and $f'(0)$ exists for all real permissible values of a and is equal to $\sqrt{5a-1-a^2}$. Then

   (a) $f(x)$ is positive for all real x
   (b) $f(x)$ is negative for all real x
   (c) $f(x) = 0$ has real roots
   (d) nothing can be said about the sign of $f(x)$

6. Let $f(3) = 4$ and $f'(3) = 5$. Then $\lim_{x \to 3} [f(x)]$ (where $[.]$ denotes the greatest integer function) is

   (a) 3 (b) 4
   (c) 5 (d) non-existent

7. Let $f(x)$ be a function which is differentiable any number of times and $f(2x^2 - 1) = 2x^3 f(x)$, $\forall x \in R$. Then $f^{(2010)}(0) =$ (Here $f^{(n)}(x) = n^{th}$ order derivative of f at x)

   (a) −1 (b) 1
   (c) 0 (d) data is insufficient

8. If $f(x) = \begin{vmatrix} (x-a)^4 & (x-a)^3 & 1 \\ (x-b)^4 & (x-b)^3 & 1 \\ (x-c)^4 & (x-c)^3 & 1 \end{vmatrix}$ then

   $f'(x) = \lambda \cdot \begin{vmatrix} (x-a)^4 & (x-a)^2 & 1 \\ (x-b)^4 & (x-b)^2 & 1 \\ (x-c)^4 & (x-c)^2 & 1 \end{vmatrix}$. The value of λ is

   (a) 1 (b) 2 (c) 3 (d) none of these

9. Suppose $\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x) \end{vmatrix} = 0$ where $f(x)$ is continuously differentiable function with $f'(x) \neq 0$ and satisfies $f(0) = 1$ and $f'(0) = 2$ then $\lim_{x \to 0} \frac{f(x)-1}{x}$ is

   (a) 1 (b) 2 (c) 1/2 (d) 0

10. A non zero polynomial with real coefficients has the property that $f(x) = f'(x)\, f''(x)$. If a is the coefficient of highest degree term of $f(x)$, then the value of a is

    (a) 1/3 (b) 6 (c) 12 (d) 1/18

11. If 'f' is an increasing function from $R \to R$ such that $f''(x) > 0$ and f^{-1} exists then $\frac{d^2(f^{-1}(x))}{dx^2}$ is

    (a) < 0 (b) > 0
    (c) $= 0$ (d) cannot be determined

12. Vertices of a variable acute angled triangle ABC lies on a fixed circle. Also, a, b, c and A, B, C are lengths of sides and angles of triangle ABC, respectively. If x_1, x_2 and x_3 are distances of orthocentre from A, B and C, respectively then the maximum value of $\left(\frac{dx_1}{da} + \frac{dx_2}{db} + \frac{dx_3}{dc}\right)$ is

    (a) $-\sqrt{3}$ (b) $-3\sqrt{3}$ (c) $\sqrt{3}$ (d) $3\sqrt{3}$

13. In a question a student was given to find the derivative of the product of two functions f and g. The student by mistake thought $(fg)' = f'g'$ for his question $f(x) = x^3$ and he got the correct answer. Given that $g(4) = 1$. Then which of the following is false?

    (a) $g(5) = \frac{1}{8}$
    (b) $f'(x) < 0$
    (c) $f(0) < 0$
    (d) None of these

Target Learning India Pvt. Ltd.

14. f is a strictly monotonic differentiable function with $f'(x) = \dfrac{1}{\sqrt{1+x^3}}$. If g is the inverse of f, then $g''(x) =$

(a) $\dfrac{3x^2}{2\sqrt{1+x^3}}$ (b) $\dfrac{3g^2(x)}{2\sqrt{1+g^2(x)}}$

(c) $\dfrac{3}{2}g^2(x)$ (d) $\dfrac{x^2}{\sqrt{1+x^3}}$

15. Suppose $f: R \to R^+$ be a differentiable function such that $3f(x+y) = f(x)f(y)\ \forall\ x, y \in R$ with $f(1) = 6$. Then the value of $f(2)$ is

(a) 6 (b) 9 (c) 12 (d) 15

Answers Key

Single Correct Answer Type

1. (d) **2.** (b) **3.** (b) **4.** (d) **5.** (a)

6. (d) **7.** (c) **8.** (c) **9.** (b) **10.** (d)

11. (a) **12.** (b) **13.** (a) **14.** (c) **15.** (c)

Cengage Learning India Pvt. Ltd.

CHAPTER

5

Applications of Derivatives

DPP 5.1

Tangent and Normal

Single Correct Answer Type

1. The equation of the normal to the curve parametrically represented by $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point $P(2, -1)$ is

(a) $2x + 3y - 1 = 0$ (b) $6x - 7y - 11 = 0$

(c) $7x + 6y - 8 = 0$ (d) $3x + y - 1 = 0$

2. If the curves $y = x^3 + ax$ and $y = bx^2 + c$ pass through the point $(-1, 0)$ and have a common tangent line at this point then the value of $a + b + c$ is

(a) 0 (b) 1 (c) -3 (d) -1

3. If the function $f(x) = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x then the value of b is equal to

(a) -2 (b) -6 (c) 6 (d) 3

4. Let $f(x) = x^3 + x + 1$ and let $g(x)$ be its inverse function then equation of the tangent to $y = g(x)$ at $x = 3$ is

(a) $x - 4y + 1 = 0$ (b) $x + 4y - 1 = 0$

(c) $4x - y + 1 = 0$ (d) $4x + y - 1 = 0$

5. A curve is represented parametrically by the equations $x = t + e^{at}$ and $y = -t + e^{at}$, $t \in R$ and $a > 0$. If the curve touches the axis of x at the point A, then the coordinates of the point A are

(a) $(1, 0)$ (b) $(2e, 0)$

(c) $(e, 0)$ (d) $(1/e, 0)$

6. The equation of the straight lines which are both tangent and normal to the curve $27x^2 = 4y^3$ are

(a) $x = \pm\sqrt{2}(y - 2)$ (b) $x = \pm\sqrt{3}(y - 2)$

(c) $x = \pm\sqrt{2}(y - 3)$ (d) $x = \pm\sqrt{3}(y - 3)$

7. If the tangent at $(1, 1)$ on $y^2 = x(2 - x)^2$ meets the curve again at P, then P is

(a) $(4, 4)$ (b) $(2, 0)$

(c) $(9/4, 3/8)$ (d) $(3, 3^{1/2})$

8. If a curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point $(0, 1)$ and also touches the x-axis at the point $(-1, 0)$ then the values of x for which the curve has a negative gradient are

(a) $x > -1$ (b) $x > 1$

(c) $x < -1$ (d) $-1 \le x \le 1$

9. From the point $(1, 1)$ tangents are drawn to the curve represented parametrically as $x = 2t - t^2$ and $y = t + t^2$. The distance between the points of contact is

(a) $\frac{2\sqrt{43}}{9}$ (b) 2 (c) 3 (d) $\frac{2\sqrt{53}}{9}$

10. The value of parameter t so that the line $(4 - t)x + ty + (a^3 - 1) = 0$ is normal to the curve $xy = 1$ may lie in the interval

(a) $(1, 4)$ (b) $(-\infty, 0) \cup (4, \infty)$

(c) $(-4, 4)$ (d) $[3, 4]$

11. The portion of the tangent at any point on the curve $x = at^3$, $y = at^4$ between the axes is divided by the abscissa of the point of contact externally in the ratio

(a) 1/4 (b) 3/4 (c) 3/2 (d) 2/5

Pearson Learning India Pvt. Ltd.

Multiple Correct Answers Type

12. Equation of a line which is tangent to both the curves $y = x^2 + 1$ and $y = -x^2$ is

(a) $y = \sqrt{2}x - \frac{1}{2}$ (b) $y = \sqrt{2}x + \frac{1}{2}$

(c) $y = -\sqrt{2}x + \frac{1}{2}$ (d) $y = -\sqrt{2}x - \frac{1}{2}$

13. For the functions defined parametrically by the equations

$$f(t) = x = \begin{cases} 2t + t^2 \sin\frac{1}{t} & ; t \neq 0 \\ 0 & ; t = 0 \end{cases} \text{ and}$$

$$g(t) = y = \begin{cases} \frac{1}{t}\sin t^2 & ; t \neq 0 \\ 0 & ; t = 0 \end{cases}$$

(a) equation of tangent at $t = 0$ is $x - 2y = 0$

(b) equation of normal at $t = 0$ is $2x + y = 0$

(c) tangent does not exist at $t = 0$

(d) normal does not exist at $t = 0$

14. For the curve $x = 2a\sin t + a \sin t \cos^2 t$; $y = -a\cos^3 t$

(a) normal is inclined at an angle $\frac{\pi}{2} + t$ with x-axis.

(b) normal is inclined at an angle t with x-axis.

(c) portion of normal contained between the co-ordinate axes is equal to $2a$.

(d) portion of normal contained between the co-ordinate axes is equal to $4a$.

15. The curve $y = ax^3 + bx^2 + cx$ is inclined at 45° to x-axis at (0, 0) but it touches x-axis at (1, 0), then

(a) $f'(1) = 0$ (b) $f''(1) = 2$

(c) $f'''(2) = 12$ (d) $f(2) = 2$

Answers Key

Single Correct Answer Type

1. (c) **2.** (d) **3.** (b) **4.** (a) **5.** (b)
6. (a) **7.** (c) **8.** (c) **9.** (d) **10.** (b)
11. (b)

Multiple Correct Answers Type

12. (b, c) **13.** (a, b) **14.** (a, c) **15.** (a, b, d)

Cengage Learning India Pvt. Ltd.

DPP 5.2

Applications of Tangent and Normal

Single Correct Answer Type

1. The length of the sub-tangent to the hyperbola $x^2 - 4y^2 = 4$ corresponding to the normal having slope unity is $\frac{1}{\sqrt{k}}$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

2. Cosine of the acute angle between the curves $y = 3^{x-1}\log_e x$ and $y = x^x - 1$ is

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

3. Acute angle between two curves $x^2 + y^2 = a^2\sqrt{2}$ and $x^2 - y^2 = a^2$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of these

4. The minimum distance between a point on the curve $y = e^x$ and a point on the curve $y = \log_e x$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 3 (d) $2\sqrt{2}$

5. Tangents are drawn from origin to the curve $y = \sin x + \cos x$. Then their points of contact lie on the curve

(a) $\frac{1}{x^2} + \frac{2}{y^2} = 1$ (b) $\frac{2}{x^2} - \frac{1}{y^2} = 1$

(c) $\frac{2}{x^2} + \frac{1}{y^2} = 1$ (d) $\frac{2}{y^2} - \frac{1}{x^2} = 1$

6. If $3x + 2y = 1$ is a tangent to $y = f(x)$ at $x = 1/2$, then $\lim_{x\to 0} \frac{x(x-1)}{f\left(\frac{e^{2x}}{2}\right) - f\left(\frac{e^{-2x}}{2}\right)}$

(a) 1/3 (b) 1/2 (c) 1/6 (d) 1/7

7. Distance of point P on the curve $y = x^{3/2}$ which is nearest to the point M (4, 0) from origin is

(a) $\sqrt{\frac{112}{27}}$ (b) $\sqrt{\frac{100}{27}}$

(c) $\sqrt{\frac{101}{9}}$ (d) $\sqrt{\frac{112}{9}}$

8. If the equation of the normal to the curve $y = f(x)$ at $x = 0$ is $3x - y + 3 = 0$ then the value of $\lim_{x\to 0} \frac{x^2}{\{f(x^2) - 5f(4x^2) + 4f(7x^2)\}}$ is

(a) –3 (b) 1/3 (c) 3 (d) –1/3

Multiple Correct Answers Type

9. If L_T, L_N, L_{ST} and L_{SN} denote the lengths of tangent, normal, sub-tangent and sub-normal, respectively, of a curve $y = f(x)$ at a point $P(2009, 2010)$ on it, then

(a) $\frac{L_{ST}}{2010} = \frac{2010}{L_{SN}}$ (b) $\left|\frac{L_T}{L_N}\sqrt{\frac{L_{SN}}{L_{ST}}}\right|$ = constant

(c) $1 - L_{ST}L_{SN} = \frac{2000}{2010}$ (d) $\left(\frac{L_T + L_N}{L_T - L_N}\right)^2 = \frac{L_{ST}}{L_{SN}}$

10. Which of the following pair(s) of family is/are orthogonal?

(a) $16x^2 + y^2 = c$ and $y^{16} = kx$

(b) $y = x + ce^{-x}$ and $x + 2 = y + ke^{-y}$

(c) $y = cx^2$ and $x^2 + 2y^2 = k$

(d) $x^2 - y^2 = c$ and $xy = k$

where c and k are arbitrary constant.

Answers Key

Single Correct Answer Type

1. (c) **2.** (b) **3.** (c) **4.** (b) **5.** (d)
6. (a) **7.** (a) **8.** (d)

Multiple Correct Answers Type

9. (a, b) **10.** (a, b, c, d)

Cengage Learning India Pvt. Ltd.

DPP 5.3

Measure of Rate

Single Correct Answer Type

1. The rate of change of $\sqrt{x^2+16}$ with respect to $\frac{x}{x-1}$ at $x = 3$ is

(a) 1 (b) $\frac{11}{5}$ (c) $-\frac{12}{5}$ (d) -3

2. The eccentricity of the ellipse $3x^2 + 4y^2 = 12$ is changed at the rate 0.1/sec. The time at which it will touch the auxiliary circle is

(a) 2 seconds (b) 3 seconds
(c) 5 seconds (d) 6 seconds

3. A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in metres) increases at a rate of 10 m/sec. If the angle of inclination θ of the line joining the particle to the origin changes, when $x = 3$ m, at the rate of k rad/sec., then the value of k is

(a) 1 (b) 2 (c) 1/2 (d) 1/3

4. If the rate of change in the volume of sphere is equal to the rate of change in its radius when the radius is $\frac{1}{k\sqrt{\pi}}$. The value of k is

(a) 1 (b) 2 (c) 0.5 (d) none of these

5. Water is dropped at the rate of 2 m^3/s into a cone of semi-vertical angle of 45°. The rate at which periphery of water surface changes when height of water in the cone is 2 m, is

(a) 0.5 m/s (b) 2 m/s
(c) 3 m/s (d) 1 m/s

6. Suppose that water is emptied from a spherical tank of radius 10 cm. If the depth of the water in the tank is 4 cm and is decreasing at the rate of 2 cm/sec, then the radius of the top surface of water is decreasing at the rate of

(a) 1 (b) 2/3 (c) 3/2 (d) 2

7. The altitude of a cone is 20 cm and its semi-vertical angle is 30°. If the semi-vertical angle is increasing at the rate of 2° per second, then the radius of the base is increasing at the rate of

(a) 30 cm/sec (b) $\frac{160}{3}$ cm/sec
(c) 10 cm/sec (d) 160 cm/sec

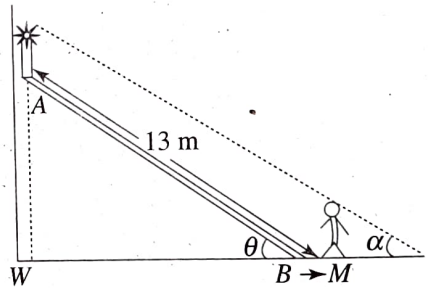
8. Let the equation of a curve be $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. If θ changes at a constant rate k then the rate of change of the slope of the tangent to the curve at $\theta = \pi/3$ is

(a) $2k/\sqrt{3}$ (b) $k/\sqrt{3}$
(c) k (d) none of these

Comprehension Type

For Questions 9 and 10

A lamp post of length 10 meter placed at the end A of a ladder AB of length 13 meters, which is leaning against a vertical wall as shown in figure and its base slides away from the wall. At the instant base B is 12 m from the vertical wall, the base B is moving at the rate of 5 m/sec. A man (M) of height 1.5 meter standing at a distance of 15 m from the vertical wall.



9. Rate at which θ decreases, when the base B is 12 m from the vertical wall, is

(a) 1 rad/sec (b) 2 rad/sec
(c) 5 rad/sec (d) 1/2 rad/sec

10. The rate at which the length of shadow of man increases, when the base B is 12 m from vertical wall, is

(a) 15 m/sec (b) 40/27 m/sec
(c) 15/2 m/sec (d) 5 m/sec

Answers Key

Single Correct Answer Type

1. (c) **2.** (c) **3.** (a) **4.** (b) **5.** (d)
6. (c) **7.** (b) **8.** (d)

Comprehension Type

9. (a) **10.** (b)

DPP 5.4

Mean Value Theorem

Subjective Type

1. Prove that for $\lambda > 1$, the equation $x \log x + x = \lambda$ has at least one solution in $[1, \lambda]$.
2. If $f(x)$ and $g(x)$ are continuous and differentiable functions, then prove that there exists $c \in [a, b]$ such that $\dfrac{f'(c)}{f(a)-f(c)} + \dfrac{g'(c)}{g(b)-g(c)} = 1$.

Single Correct Answer Type

3. Consider $f(x) = |1 - x|$, $1 \le x \le 2$ and $g(x) = f(x) + b \sin \dfrac{\pi}{2} x$, $1 \le x \le 2$ then which of the following is correct?
   (a) Rolle's theorem is applicable to both f and g with $b = \dfrac{3}{2}$.
   (b) LMVT is not applicable to f and Rolle's theorem is applicable to g with $b = \dfrac{1}{2}$.
   (c) LMVT is applicable to f and Rolle's theorem is applicable to g with $b = 1$.
   (d) Rolle's theorem is not applicable to both f and g for any real b.
4. If $c = \dfrac{1}{2}$ and $f(x) = 2x - x^2$, then interval of x in which LMVT is applicable, is
   (a) $(1, 2)$ (b) $(-1, 1)$
   (c) $(0, 1)$ (d) $(2, 1)$
5. If a twice differentiable function $f(x)$ on (a, b) and continuous on $[a, b]$ is such that $f''(x) < 0$ for all $x \in (a; b)$ then for any $c \in (a, b)$, $\dfrac{f(c)-f(a)}{f(b)-f(c)} >$
   (a) $\dfrac{b-c}{c-a}$ (b) $\dfrac{c-a}{b-c}$
   (c) $(b-c)(c-a)$ (d) $\dfrac{1}{(b-c)(c-a)}$
6. Let $a, n \in N$ such that $a \ge n^3$. Then $\sqrt[3]{a+1} - \sqrt[3]{a}$ is always
   (a) less than $\dfrac{1}{3n^2}$ (b) less than $\dfrac{1}{2n^3}$
   (c) more than $\dfrac{1}{n^3}$ (d) more than $\dfrac{1}{4n^2}$
7. Given $f'(1) = 1$ and $f(2x) = f(x)$, $\forall\ x > 0$. If $f'(x)$ is differentiable then there exists a number $c \in (2, 4)$ such that $f''(c)$ equals
   (a) 1/4 (b) −1/2 (c) −1/4 (d) −1/8

Multiple Correct Answers Type

8. Let $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 3-x & 5-3x^2 & 3x^3-1 \\ 2x^2-1 & 3x^5-1 & 7x^8-1 \end{vmatrix}$. Then which of the following is/are correct?
   (a) $f(x) = 0$ has at least two real roots
   (b) $f'(x) = 0$ has at least one real root.
   (c) $f(x)$ is many-one function
   (d) none of these
9. Which of the following is correct?
   (a) $\dfrac{\tan^{-1} x - \tan^{-1} y}{x-y} \le 1\ \forall\ x, y \in R, (x \ne y)$
   (b) $\dfrac{\sin^{-1} x - \sin^{-1} y}{x-y} > 1\ \forall\ x, y \in [-1, 1], x \ne y$
   (c) $\dfrac{\cos^{-1} x - \cos^{-1} y}{y-x} < 1\ \forall\ x, y \in [-1, 1], x \ne y$
   (d) $\dfrac{\cot^{-1} x - \cot^{-1} y}{y-x} > 1, \forall\ x, y \in R, x \ne y$

Comprehension Type

For Questions 10–12

Let $f(x)$ be a function such that its derivative $f'(x)$ is continuous in $[a, b]$ and differentiable in (a, b). Consider a function $\phi(x) = f(b) - f(x) - (b - x) f'(x) - (b - x)^2 A$. If Rolle's theorem is applicable to $\phi(x)$ on $[a, b]$, answer following questions.

10. If there exists some number $c (a < c < b)$ such that $\phi'(c) = 0$ and $f(b) = f(a) + (b - a) f'(a) + \lambda(b - a)^2 f''(c)$, then λ is

Cengage Learning India Pvt. Ltd.

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

11. Let $f(x) = x^3 - 3x + 3$, $a = 1$ and $b = 1 + h$. If there exists $c \in (1, 1 + h)$ such that $\phi'(c) = 0$ and $\frac{f(1+h) - f(1)}{h^2} = \lambda c$, then $\lambda =$

(a) 1/2 (b) 2
(c) 3 (d) does not exist

12. Let $f(x) = \sin x$, $a = \alpha$ and $b = \alpha + h$. If there exists a real number t such that $0 < t < 1$, $\phi'(\alpha + th) = 0$ and $\frac{\sin(\alpha + h) - \sin\alpha - h\cos\alpha}{h^2} = \lambda \sin(\alpha + th)$, then $\lambda =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Answers Key

Subjective Type

2. 1

Single Correct Answer Type

3. (c) **4.** (c) **5.** (b) **6.** (a) **7.** (d)

Multiple Correct Answers Type

8. (a, b, c) **9.** (a, b)

Comprehension Type

10. (c) **11.** (c) **12.** (b)

Cengage Learning India Pvt. Ltd.

6 Monotonocity and Maxima-Minima of Functions

DPP 6.1

Monotonocity and Its Applications (Part I)

Single Correct Answer Type

1. If $x \in (0, \pi/2)$, then the function $f(x) = x \sin x + \cos x + \cos^2 x$ is
   (a) Increasing
   (b) Decreasing
   (c) Neither increasing nor decreasing
   (d) None of these
2. The function $f : [a, \infty) \to R$ where R denotes the range corresponding to the given domain, with rule $f(x) = 2x^3 - 3x^2 + 6$, will have an inverse provided
   (a) $a \le 1$ (b) $a \ge 0$ (c) $a \le 0$ (d) $a \ge 1$
3. Let $f(x) = 1 - x - x^3$. In which of the following intervals, inequality $1 - f(x) - f^3(x) > f(1 - 5x)$ is not satisfied?
   (a) $(-2, 0)$ (b) $(2, \infty)$
   (c) $(0, 2)$ (d) none of these
4. If $g(x) = 2f(2x^3 - 3x^2) + f(6x^2 - 4x^3 - 3)$, $\forall\ x \in R$ and $f'(x) > 0$, $\forall\ x \in R$, then $g(x)$ is increasing for x belonging to
   (a) $\left(-\infty, -\frac{1}{2}\right) \cup (0, 1)$ (b) $\left(-\frac{1}{2}, 0\right) \cup (1, \infty)$
   (c) $(0, \infty)$ (d) $(-\infty, 1)$
5. If $f(x) = \sin 2x - 8(a + 1) \sin x + (4a^2 + 8a - 14)x$ increases for all $x \in R$ and has no critical points for all $x \in R$, then which of the following values of a are not possible?
   (a) $(-\infty, -\sqrt{5} - 2)$ (b) $(1, \infty)$
   (c) $(\sqrt{5}, \infty)$ (d) none of these
6. The minimum value of k for which $f(x) = 2e^x - ke^{-x} + (2k + 1)x - 3$ is monotonically increasing for all real x is
   (a) 2 (b) 1 (c) 0 (d) -1
7. If the function $f(x) = 2 \cot x + (2a + 1)\log_e|\text{cosec } x| + (2 - a)x$ is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$, then range of '$a$' is
   (a) $[0, \infty)$ (b) $(-\infty, 0]$
   (c) $(-\infty, \infty)$ (d) none of these

Multiple Correct Answers Type

8. Which of the following statement(s) is/are true?
   (a) Differentiable function satisfying $f(-1) = f(1)$ and $f'(x) \ge 0$ for all x must be a constant function on the interval $[-1, 1]$.
   (b) There exists a function with domain R satisfying $f(x) < 0$ for all x, $f'(x) > 0$ for all x and $f''(x) > 0$ for all x.
   (c) If $f''(c) = 0$ then $(c, f(c))$ is an inflection point.
   (d) Suppose $f(x)$ is a function whose derivative is the function $f'(x) = 2x^2 + 2x - 12$. Then $f(x)$ is decreasing for $-3 < x < 2$ and concave up for $x > -\frac{1}{2}$.
9. Let $f : R \to R$, $f(x) = x + \log_e(1 + x^2)$. Then
   (a) f is injective
   (b) f is surjective
   (c) there is a point on the graph of $y = f(x)$ where tangent is not parallel to any of the chords
   (d) inverse of $f(x)$ exists.

Cengage Learning India Pvt. Ltd.

10. Let $f(x) = x - \frac{1}{x}$ then which one of the following statements is true?

(a) $f(x)$ is one-one function.
(b) $f(x)$ is increasing function.
(c) $f(x) = k$ has two distinct real roots for any real k.
(d) $x = 0$ is point of inflection.

11. Let $f(x)$ be an even function in R. If $f(x)$ is monotonically increasing in $[2, 6]$, then

(a) $f(3) < f(-5)$ (b) $f(4) < f(-3)$
(c) $f(2) > f(-3)$ (d) $f(-3) < f(5)$

12. If $f(x) = \begin{cases} -e^{-x} + k & , \quad x \le 0 \\ e^{x} + 1 & , \quad 0 < x < 1 \\ ex^2 + \lambda & , \quad x \ge 1 \end{cases}$ is one-one a

monotonically increasing $\forall\, x \in R$, then

(a) maximum value of k is 1
(b) maximum value of k is 3
(c) minimum value of λ is 0
(d) minimum value of λ is 1

Answers Key

Single Correct Answer Type

1. (b) **2.** (d) **3.** (c) **4.** (b) **5.** (b)
6. (c) **7.** (a)

Multiple Correct Answers Type

8. (a, d) **9.** (a, b, c, d)
10. (b, c, d) **11.** (a, d) **12.** (b, d)

Cengage Learning India Pvt. Ltd

Monotonocity and Its Applications (Part II)

Single Correct Answer Type

1. If $x_1, x_2 \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\tan x_2}{\tan x_1}$ is (where $x_1 < x_2$)

(a) $< \frac{x_1}{x_2}$ (b) $= \frac{x_1}{x_2}$

(c) $< x_1 x_2$ (d) $> \frac{x_2}{x_1}$

2. If $f(x)$ is a differentiable real valued function satisfying $f''(x) - 3f'(x) > 3 \ \forall \ x \geq 0$ and $f'(0) = -1$, then $f(x) + x \ \forall \ x > 0$ is

(a) decreasing function of x
(b) increasing function of x
(c) constant function
(d) none of these

3. The roots of $(x-41)^{49} + (x-49)^{41} + (x-2009)^{2009} = 0$ are

(a) all necessarily real
(b) non-real except one positive real root
(c) non-real except three positive real roots
(d) non-real except for three real roots of which exactly one is positive

4. Let h be a twice continuously differentiable positive function on an open interval H. Let $g(x) = \log_e(h(x))$ for each $x \in H$
Suppose $(h'(x))^2 > h''(x)h(x)$ for each $x \in H$.
Then, we can conclude that

(a) g is increasing on H (b) g is decreasing on H
(c) g is concave up on H (d) g is concave down on H

5. If $\sin x + x \geq |k|x^2, \ \forall x \in \left[0, \frac{\pi}{2}\right]$, then the greatest value of k is

(a) $\frac{-2(2+\pi)}{\pi^2}$

(b) $\frac{2(2+\pi)}{\pi^2}$

(c) can't be determined finitely
(d) zero

6. If $4x + 8\cos x + \tan x - 2\sec x - 4\log\{\cos x(1+\sin x)\} \geq 6$ for all $x \in [0, \lambda)$ then the largest value of λ is

(a) $\pi/3$ (b) $\pi/6$ (c) $\pi/4$ (d) $3\pi/4$

Comprehension Type

For Questions 7–9

Let $f: R \to R$, $y = f(x)$, $f(0) = 0$, $f'(x) > 0$ and $f''(x) > 0$. Three points $A(\alpha, f(\alpha))$, $B(\beta, f(\beta))$, $C(\gamma, f(\gamma))$ on $y = f(x)$ such that $0 < \alpha < \beta < \gamma$.

7. Which of the following is false?

(a) $\alpha f(\beta) > \beta(f(\alpha))$ (b) $\alpha f(\beta) < \beta f(\alpha)$
(c) $\gamma f(\beta) < \beta(f(\gamma))$ (d) $\gamma(f(\alpha)) < \alpha f(\gamma)$

8. Which of following is true?

(a) $\frac{f(\alpha)+f(\beta)}{2} < f\left(\frac{\alpha+\beta}{2}\right)$

(b) $\frac{f(\alpha)+f(\beta)}{2} > f\left(\frac{\alpha+\beta}{2}\right)$

(c) $\frac{f(\alpha)+f(\beta)}{2} = f\left(\frac{\alpha+\beta}{2}\right)$

(d) $\frac{2f(\alpha)+f(\beta)}{3} < f\left(\frac{2\alpha+\beta}{3}\right)$

9. Which of the following is true?

(a) $\gamma f(\gamma+\beta-\alpha) > (\gamma+\beta-\alpha) f(\gamma)$
(b) $\gamma f(\gamma+\beta-\alpha) < (\gamma+\beta-\alpha) f(\gamma)$
(c) $\alpha f(\gamma+\beta-\alpha) < (\gamma+\beta-\alpha) f(\alpha)$
(d) none of these

For Questions 10–12

Let f be a twice differentiable function such that $f''(x) > 0 \ \forall \ x \in R$. Let $h(x)$ is defined by $h(x) = f(\sin^2 x) + f(\cos^2 x)$ where $|x| < \frac{\pi}{2}$.

10. The number of critical points of $h(x)$ are

(a) 1 (b) 2
(c) 3 (d) more than 3

Cengage Learning India Pvt. Ltd.

11. $f'(\sin^2 x) < f'(\cos^2 x)$ for $x \in$

(a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (b) $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

12. $h(x)$ is increasing for $x \in$

(a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (b) $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(0, \frac{\pi}{4}\right)$

Answers Key

Single Correct Answer Type

1. (d) **2.** (b) **3.** (b) **4.** (d) **5.** (b)
6. (b)

Comprehension Type

7. (b) **8.** (b) **9.** (a) **10.** (c) **11.** (a)
12. (b)

Cengage Learning India Pvt. Ltd.

DPP 6.3

Extrema and Its Applications (Part I)

Single Correct Answer Type

1. The greatest possible value of the expression $\tan x + \cot x + \cos x$ on the interval $[\pi/6, \pi/4]$ is

(a) $\frac{12}{5}\sqrt{2}$ (b) $\frac{11}{6}\sqrt{2}$ (c) $\frac{12}{5}\sqrt{3}$ (d) $\frac{11}{6}\sqrt{3}$

2. Let $f(x)=\begin{cases}(x+1)^3 & -2<x\le -1\\ x^{2/3}-1 & -1<x\le 1\\ -(x-1)^2 & 1<x<2\end{cases}$. The total number of maxima and minima of $f(x)$ is

(a) 4 (b) 3 (c) 2 (d) 1

3. Consider the graph of the function $f(x) = x + \sqrt{|x|}$

Statement 1: The graph of $y = f(x)$ has only one critical point.

Statement 2: $f'(x)$ vanishes only at one point.

(a) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is false, Statement 2 is true.

4. The minimum value of $\frac{\tan\left(x+\frac{\pi}{6}\right)}{\tan x}, x\in\left(0,\frac{\pi}{3}\right)$, is

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 3

5. Let $f(x)=\frac{x^2+2}{[x]}, 1\le x\le 3$, where [.] is the greatest integer function. Then the least value of $f(x)$ is

(a) 2 (b) 3 (c) 3/2 (d) 1

6. If $f(x)=\begin{cases}3-x^2, & x\le 2\\ \sqrt{a+14}-|x-48|, & x>2\end{cases}$ and if $f(x)$ has a local maxima at $x = 2$, then greatest value of a is

(a) 2013 (b) 2012 (c) 2011 (d) 2010

7. The function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ has

(a) One minima and two maxima
(b) Two minima and one maxima
(c) Two minima and two maxima
(d) One minima and one maxima

8. If $f(x) = |x - 1| + |x + 4| + |x - 9| + \ldots + |x - 2500|\ \forall\ x \in R$, then all the values of x where $f(x)$ has minimum values lie in

(a) (600, 700) (b) (576, 678)
(c) (625, 678) (d) none of these

9. Slope of tangent to the curve $y = 2e^x \sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)$, where $0 \le x \le 2\pi$ is minimum at $x =$

(a) 0 (b) π
(c) 2π (d) none of these

10. The value of a for which all extremum of function $f(x) = x^3 - 3ax^2 + 3(a^2 - 1)x + 1$, lie in the interval (2, 4) is

(a) (3, 4) (b) (–1, 3)
(c) (–3, –1) (d) none of these

Multiple Correct Answers Type

11. If the function $f(x) = ax\, e^{-bx}$ has a local maximum at the point (2, 10), then

(a) $a = 5e$ (b) $a = 5$,
(c) $b = 1$ (d) $b = 1/2$

12. Let $f(x)=\frac{e^x}{1+x^2}$ and $g(x) = f'(x)$, then

(a) $g(x)$ has two local maxima and two local minima points
(b) $g(x)$ has exactly one local maxima and one local minima point
(c) $x = 1$ is a point of local maxima for $g(x)$
(d) There is a point of local maxima for $g(x)$ in the interval (–1, 0)

13. If $f'(x) = (x - a)^{2010}(x - b)^{2009}$ and $a > b$, then

(a) $f(x)$ has relative maxima at $x = b$
(b) $f(x)$ has relative minima at $x = b$
(c) $f(x)$ has relative maxima at $x = a$
(d) $f(x)$ has neither maxima, nor minima at $x = a$

14. If $\lim_{x\to a} f(x) = \lim_{x\to a} [f(x)]$ ([.] denotes the greatest integer function) and $f(x)$ is non–constant continuous function, then

(a) $\lim_{x\to a} f(x)$ is an integer
(b) $\lim_{x\to a} f(x)$ is non-integer

Cengage Learning India Pvt. Ltd.

(c) $f(x)$ has local maximum at $x = a$
(d) $f(x)$ has local minimum at $x = a$

15. Consider the function $f(x) = \log_e(\sqrt{1-x^2} - x)$. Then which of the following is/are true?

(a) $f(x)$ increases in the on $x = \left(-1, -\frac{1}{\sqrt{2}}\right)$

(b) f has local maximum at $x = -\frac{1}{\sqrt{2}}$

(c) Least value of f does not exist

(d) Least value of f exists

Answers Key

Single Correct Answer Type

1. (d) **2.** (b) **3.** (d) **4.** (d) **5.** (b)
6. (c) **7.** (d) **8.** (c) **9.** (b) **10.** (b)

Multiple Correct Answers Type

11. (a, d) **12.** (b, d) **13.** (b, d) **14.** (a, d)
15. (a, b, c)

Cengage Learning India Pvt. Ltd.

DPP 6.4

Extrema and Its Applications (Part II)

Single Correct Answer Type

1. If $f(x) = \begin{cases} x^3(1-x), & x \le 0 \\ x\log_e x + 3x, & x > 0 \end{cases}$, then which of the following is not true?

(a) $f(x)$ has point of maxima at $x = 0$

(b) $f(x)$ has point minima at $x = e^{-4}$

(c) $f(x)$ has range R

(d) None of these

2. The coordinates of the point on the curve $x^3 = y(x-a)^2$ where the ordinate is minimum is

(a) $\left(3a, \frac{27}{4}a\right)$ (b) $(2a, 8a)$

(c) $(a, 0)$ (d) None of these

3. The fraction exceeding its pth power by the greatest number possible, where $p \ge 2$, is

(a) $\left(\frac{1}{p}\right)^{1/(p-1)}$ (b) $\left(\frac{1}{p}\right)^{p-1}$

(c) $p^{1/p-1}$ (d) none of these

4. If $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - e^{x-1}, & 1 < x \le 2 \\ x - e, & 2 < x \le 3 \end{cases}$ and $g'(x) = f(x)$, $x \in [1, 3]$, then

(a) $g(x)$ has no local maxima

(b) $g(x)$ has no local minima

(c) $g(x)$ has local maxima at $x = 1 + \ln 2$ and local minima at $x = e$

(d) $g(x)$ has local minima at $x = 1 + \ln 2$ and local maxima at $x = e$

5. If $g(x) = \max(y^2 - xy)$ $(0 \le y \le 1)$, then the minimum value of $g(x)$ (for real x) is

(a) $\frac{1}{4}$ (b) $3 - \sqrt{3}$ (c) $3 + \sqrt{8}$ (d) $\frac{1}{2}$

6. If $a, b \in R$ distinct numbers satisfying $|a-1| + |b-1| = |a| + |b| = |a+1| + |b+1|$, then the minimum value of $|a-b|$ is

(a) 3 (b) 0 (c) 1 (d) 2

7. If the equation $2x^3 - 6x + 2\sin a + 3 = 0$, $a \in (0, \pi)$ has only one real root, then the largest interval in which a lies is

(a) $\left(0, \frac{\pi}{6}\right)$ (b) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(c) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{5\pi}{6}, \pi\right)$

8. Let f be a continuous and differentiable function in (x_1, x_2). If $f(x).f'(x) \ge x\sqrt{1-(f(x))^4}$ and $\lim\limits_{x \to x_1^+}(f(x))^2 = 1$ and $\lim\limits_{x \to x_2^-}(f(x))^2 = \frac{1}{2}$, then minimum value of $(x_1^2 - x_2^2)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$ (d) None of these

9. If $ab = 2a + 3b$, $a > 0$, $b > 0$, then the minimum value of ab is

(a) 12 (b) 24

(c) $\frac{1}{4}$ (d) None of these

10. Let a, b, c, d, e, f, g, h be distinct elements in the set $\{-7, -5, -3, -2, 2, 4, 6, 13\}$. The minimum value of $(a+b+c+d)^2 + (e+f+g+h)^2$ is

(a) 30 (b) 32 (c) 34 (d) 40

11. The perimeter of a sector is p. The area of the sector is maximum when its radius is

(a) $\sqrt{p}$ (b) $\frac{1}{\sqrt{p}}$ (c) $\frac{p}{2}$ (d) $\frac{p}{4}$

12. Minimum integral value of k for which the equation $e^x = kx^2$ has exactly three real distinct solutions,

(a) 1 (b) 2 (c) 3 (d) 4

13. Let $f(x) = x^3 - 3x + 1$. Then number of different real solutions of $f(f(x)) = 0$

(a) 2 (b) 4 (c) 5 (d) 7

Answers Key

Single Correct Answer Type

1. (d) **2.** (a) **3.** (a) **4.** (c) **5.** (b)
6. (d) **7.** (c) **8.** (c) **9.** (b) **10.** (b)
11. (d) **12.** (b) **13.** (d)

Cengage Learning India Pvt. Ltd.

CHAPTER

7

Indefinite Integration

DPP 7.1

Integration of Elementary Functions

Single Correct Answer Type

1. $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} dx$ is equal to

(a) $-\left(\frac{\sin 2x}{2} + \cos x\right) + C$

(b) $-\left(\frac{\cos 2x}{2} + \sin x\right) + C$

(c) $-\left(\frac{\cos 2x}{2} + \cos x\right) + C$

(d) $-\left(\frac{\sin 2x}{2} + \sin x\right) + C$

2. $\int \frac{\sec x \cdot \operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x} dx$ is equal to

(a) $\frac{1}{2} \ln |\sec 2x + \tan 2x| + C$

(b) $\ln |\sec x + \operatorname{cosec} x| + C$

(c) $\ln |\sec x + \tan x| + C$

(d) $\frac{1}{2} \ln |\sec x + \operatorname{cosec} x| + C$

3. $\int \frac{1}{x} \ln\left(\frac{x}{e^x}\right) dx =$

(a) $\frac{1}{2}e^x - \ln x + C$ (b) $\frac{1}{2}\ln x - e^x + C$

(c) $\frac{1}{2}(\ln x)^2 - x + C$ (d) $\frac{e^x}{2x} + C$

4. $\int \frac{\cos^{n-1} x}{\sin^{n+1} x} dx, n \neq 0$ is

(a) $\frac{\cot^n x}{n}$ (b) $\frac{-\cot^{n-1} x}{n-1}$

(c) $\frac{-\cot^n x}{n}$ (d) $\frac{\cot^{n-1} x}{n-1}$

5. If $\int x^{26} \cdot (x-1)^{17} \cdot (5x-3) dx = \frac{x^{27} \cdot (x-1)^{18}}{k} + C$ where C is a constant of integration, then the value of k is equal to

(a) 3 (b) 6 (c) 9 (d) 12

6. If $\int \frac{x + (\cos^{-1} 3x)^2}{\sqrt{1-9x^2}} dx = A\sqrt{1-9x^2} + B(\cos^{-1} 3x)^3 + C$, then $A - B$ is

(a) $\frac{2}{9}$ (b) $-\frac{1}{9}$ (c) $\frac{1}{9}$ (d) 0

7. If $\int \tan^9 x \, dx = f(x) + \log|\cos x|$, where $f(x)$ is a polynomial of degree n in $\tan x$, then the value of n is

(a) 6 (b) 7 (c) 8 (d) none of these

Cengage Learning India Pvt. Ltd.

8. $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x} dx = \log_e(f(x)) + g(x) + C$ where C is the constant of integration and $f(x)$ is positive. Then $f(x) + g(x)$ has the value equal to

(a) $e^x + \sin x + 2x$ (b) $e^x + \sin x$

(c) $e^x - \sin x$ (d) $e^x + \sin x + x$

9. $\int \frac{x + x^{\frac{2}{3}} + x^{\frac{1}{6}}}{x\left(1 + x^{\frac{1}{3}}\right)} dx$ equals

(a) $\frac{3x^{\frac{2}{3}}}{4} + 6\tan^{-1}\left(x^{\frac{1}{6}}\right) + C$

(b) $\frac{3x^{\frac{2}{3}}}{2} + 6\tan^{-1}\left(x^{\frac{1}{6}}\right) + C$

(c) $\frac{3x^{\frac{2}{3}}}{10} + 6\tan^{-1}\left(x^{\frac{1}{6}}\right) + C$

(d) $\frac{3x^{\frac{2}{3}}}{5} + 6\tan^{-1}\left(x^{\frac{1}{6}}\right) + C$

10. $\int \frac{e^x(x-2)}{x(x^2+e^x)} dx \ \forall\, x > 0$ is equal to

(a) $\ln\left(1 + \frac{e^x}{x^2}\right) + c$ (b) $\ln\left(-\frac{1}{2} + \frac{e^x}{x^2}\right) + c$

(c) $\ln\left(2 + \frac{e^x}{x^2}\right) + c$ (d) $\ln\left(x + \frac{e^x}{x^2}\right) + c$

Answers Key

Single Correct Answer Type

1. (d) **2.** (a) **3.** (c) **4.** (c) **5.** (c) **6.** (d) **7.** (c) **8.** (b) **9.** (b) **10.** (a)

Cengage Learning India Pvt. Ltd.

DPP 7.2

Integration by Elementary Substitution

Single Correct Answer Type

1. Let $x^2 \neq n\pi - 1$, $n \in N$, then

$$\int x\sqrt{\frac{2\sin(x^2+1)-\sin 2(x^2+1)}{2\sin(x^2+1)+\sin 2(x^2+1)}}\,dx \text{ is equal to}$$

(a) $\ln\left|\frac{1}{2}\sec(x^2+1)\right|+C$

(b) $\ln\left|\sec\left(\frac{x^2+1}{2}\right)\right|+C$

(c) $\frac{1}{2}\ln|\sec(x^2+1)|+C$

(d) $\frac{1}{2}\ln\left|\frac{2}{\sec(x^2+1)}\right|+C$

2. The value of $\int\frac{\operatorname{cosec} x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}dx$ is

(a) $-\tan\left(1+\log\tan\frac{x}{2}\right)+c$

(b) $\sec^2\left(1+\log\tan\frac{x}{2}\right)+c$

(c) $\tan\left(1+\log\tan\frac{x}{2}\right)+c$

(d) $\sin^2\left(1+\log\tan\frac{x}{2}\right)+c$

3. $\int\frac{dx}{x\sqrt{x^6-16}}=$

(a) $\sec\frac{x^3}{4}+c$ (b) $\frac{1}{12}\sec\frac{x^3}{4}+c$

(c) $\frac{1}{3}\sec\frac{x^3}{4}+c$ (d) none of these

4. $\int\frac{dx}{\cos(2x)\cos(4x)}=$

(a) $\frac{1}{2\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin 2x}{1-\sqrt{2}\sin 2x}\right|-\frac{1}{2}(\log|\sec 2x-\tan 2x|)+C$

(b) $\frac{1}{2\sqrt{2}}\log\left|\frac{1-\sqrt{2}\sin 2x}{1+\sqrt{2}\sin x}\right|-\frac{1}{2}(\log|\sec 2x-\tan 2x|)+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin 2x}{1-\sqrt{2}\sin 2x}\right|-\frac{1}{2}(\log|\sec 2x-\tan 2x|)+C$

(d) none of these

5. $\int x\,2^{\ln(x^2+1)}dx$ is equal to

(a) $\frac{2^{\ln(x^2+1)}}{2(x^2+1)}+C$ (b) $\frac{(x^2+1)2^{\ln(x^2+1)}}{\ln 2+1}+C$

(c) $\frac{(x^2+1)^{\ln 2+1}}{2(\ln 2+1)}+C$ (d) $\frac{(x^2+1)^{\ln 2}}{2(\ln 2+1)}+C$

6. If $\int\frac{\sin x}{\sin\left(x-\frac{\pi}{4}\right)}dx = Af(x)+\frac{1}{\sqrt{2}}\log|\sin x-\cos x|+c$, then

(a) $A=\frac{1}{\sqrt{2}}, f(x)=\sin x$

(b) $A=\sqrt{2}, f(x)=\cos x$

(c) $A=\sqrt{2}, f(x)=x$

(d) $A=\frac{1}{\sqrt{2}}, f(x)=x$

7. $\int\left(\sqrt{\frac{\cos x}{x}}-\sqrt{\frac{x}{\cos x}}\cdot\sin x\right)dx$ equals

(a) $-\sqrt{x\cos x}+C$ (b) $\sqrt{x\sin x}+C$

(c) $2\sqrt{x\cos x}+C$ (d) $C-2\sqrt{x\cos x}$

8. $\int\frac{(2x+1)}{(x^2+4x+1)^{3/2}}dx$

(a) $\frac{x^3}{(x^2+4x+1)^{1/2}}+C$ (b) $\frac{x}{(x^2+4x+1)^{1/2}}+C$

(c) $\frac{x^2}{(x^2+4x+1)^{1/2}}+C$ (d) $\frac{1}{(x^2+4x+1)^{1/2}}+C$

Cengage Learning India Pvt. Ltd.

9. If $\int \frac{(2x+3)\,dx}{x(x+1)(x+2)(x+3)+1} = C - \frac{1}{f(x)}$ where $f(x)$ is of the form of $ax^2 + bx + c$, then the value of $f(1)$ is

(a) 4 (b) 5 (c) 6 (d) none

10. The integral $\int \sqrt{\cot x}\, e^{\sqrt{\sin x}} \sqrt{\cos x}\, dx$ equals

(a) $\frac{\sqrt{\tan x}\, e^{\sqrt{\sin x}}}{\sqrt{\cos x}} + C$ (b) $2e^{\sqrt{\sin x}} + C$

(c) $-\frac{1}{2} e^{\sqrt{\sin x}} + C$ (d) $\frac{\sqrt{\cot x}\, e^{\sqrt{\sin x}}}{2\sqrt{\cos x}} + C$

11. $\int \frac{dx}{x\sqrt{x^6+1}}$ equals

(a) $\sec^{-1} x^3 + C$

(b) $\frac{1}{6}\log\left(\frac{\sqrt{x^6+1}-1}{\sqrt{x^6+1}+1}\right) + C$

(c) $\frac{1}{3}\log\left(\frac{\sqrt{x^3+1}-1}{\sqrt{x^3+1}+1}\right) + C$

(d) $\frac{1}{3}\log\left(\frac{\sqrt{x^6+1}+1}{\sqrt{x^6+1}-1}\right) + C$

12. $\int \frac{dx}{(1+\sqrt{x})^{2010}} = 2\left[\frac{1}{\alpha(1+\sqrt{x})^{\alpha}} - \frac{1}{\beta(1+\sqrt{x})^{\beta}}\right] + c$ where $\alpha, \beta > 0$ then $\alpha - \beta$ is

(a) 1 (b) 2 (c) –1 (d) –2

13. $\int \frac{\sin\left(\frac{\pi}{4} - x\right)dx}{2 + \sin 2x} = A \tan^{-1}(f(x)) + B$, where A, B are constants. Then the range of $Af(x)$ is

(a) $[-1, 1]$ (b) $[-\sqrt{2}, \sqrt{2}]$

(c) $[0, 1]$ (d) $[-1, 0]$

Answers Key

Single Correct Answer Type

1. (b) **2.** (c) **3.** (b) **4.** (b) **5.** (c) **6.** (d) **7.** (c) **8.** (b) **9.** (b) **10.** (b) **11.** (b) **12.** (a) **13.** (a)

Cengage Learning India Pvt. Ltd.

DPP 7.3

Integration by Typical Substitution

Single Correct Answer Type

1. If $\int \sqrt{x+\sqrt{x^2+2}}\,dx = A\{x+\sqrt{x^2+2}\}^{3/2} + \frac{B}{\sqrt{x+\sqrt{x^2+2}}} + c$,

then the value of $3AB$ is

(a) -1 (b) -2 (c) 1 (d) 2

2. $\int \frac{x^2+1}{x\sqrt{x^2+2x-1}\sqrt{1-x^2-x}}dx$ is equal to

(a) $2\sin^{-1}\sqrt{x-\frac{1}{x}+2}+c$

(b) $2\cos^{-1}\sqrt{x-\frac{1}{x}+2}+c$

(c) $\sin^{-1}\sqrt{x-\frac{1}{x}+2}+c$

(d) $\cos^{-1}\sqrt{x-\frac{1}{x}+2}+c$

3. $\int \frac{dx}{x^2\sqrt{16-x^2}}$ has the value equal to

(a) $C-\frac{1}{4}\tan^{-1}\sec\left(\frac{x}{4}\right)$ (b) $\frac{1}{4}\tan^{-1}\sec\left(\frac{x}{4}\right)+C$

(c) $C-\frac{\sqrt{16-x^2}}{16x}$ (d) $\frac{\sqrt{16-x^2}}{16x}+C$

4. If $\int \frac{3\tan\left(x-\frac{\pi}{4}\right)}{\cos^2 x\sqrt{\tan^3 x+\tan^2 x+\tan x}} = K\tan^{-1}(\sqrt{\tan x+1+\cot x})+C$, then the value of K is

(a) 2 (b) 3 (c) 6 (d) 8

5. $\int \frac{\sqrt{1-x^2}-x}{\sqrt{1-x^2}(1+x\sqrt{1-x^2})}dx$ is

(a) $2\tan^{-1}(x+\sqrt{1-x^2})+c$

(b) $\tan^{-1}(x+\sqrt{1-x^2})+c$

(c) $2\tan^{-1}(x-\sqrt{1-x^2})+c$

(d) $2\cot^{-1}(x+\sqrt{1-x^2})+c$

6. $\int \frac{3x^2+2x}{x^6+2x^5+x^4+2x^3+2x^2+5}dx =$

(a) $\frac{1}{4}\tan^{-1}\left(\frac{x^3+x^2+1}{2}\right)+c$

(b) $\frac{1}{2}\tan^{-1}\left(\frac{x^3+x^2+1}{2}\right)+c$

(c) $\sin^{-1}\left(\frac{x^3+x^2+1}{2}\right)+c$

(d) $\frac{1}{2}\tan^{-1}\left(\frac{x^3+x^2}{2}\right)+c$

7. $\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}dx$ is equal to

(a) $2\left(\sqrt{\frac{x}{\sqrt{1-x}}}-\frac{1}{\sqrt{1-x}}\right)+c$

(b) $2\left(\sqrt{\frac{x}{\sqrt{1-x}}}-\frac{1}{1-x}\right)+c$

(c) $2\left(\sqrt{\frac{x}{1-x}}-\frac{1}{\sqrt{1-x}}\right)+c$

(d) $2\left(\sqrt{\frac{x}{1-x}}-\frac{1}{1-x}\right)+c$

8. If $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx$, and $f(0)=0$, then the value of $f(1)$ is

(a) $-1/2$ (b) $1/4$ (c) $1/2$ (d) $-1/4$

Cengage Learning India Pvt. Ltd.

9. If $I = \int \frac{dx}{x^4\sqrt{a^2+x^2}}$, then I equals

(a) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{3x^2}\sqrt{a^2+x^2}\right] + c$

(b) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{2\sqrt{x}}(a^2+x^2)^{3/2}\right] + c$

(c) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{3x^3}(a^2+x^2)^{3/2}\right] + c$

(d) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} + \frac{1}{3x^3}\sqrt{a^2+x^2}\right] + c$

10. $\int x^{27}(1+x+x^2)^6(6x^2+5x+4)\,dx =$

(a) $\frac{x^4(1+x+x^2)^7}{7} + C$

(b) $\frac{x^{28}(1+x+x^2)^7}{7} + C$

(c) $\frac{x^{28}(1+x+x^2)^7}{28} + C$

(d) None

11. $\int \frac{x^2(1-\log x)}{(\log x)^4 - x^4}dx$ equals

(a) $\frac{1}{2}\ln\left(\frac{x}{\ln x}\right) - \frac{1}{4}\ln(\ln^2 x - x^2) + C$

(b) $\frac{1}{4}\ln\left(\frac{\ln x - x}{\ln x + x}\right) - \frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right) + C$

(c) $\frac{1}{4}\ln\left(\frac{\ln x + x}{\ln x - x}\right) + \frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right) + C$

(d) $\frac{1}{4}\left(\ln\left(\frac{\ln x - x}{\ln x + x}\right) + \tan^{-1}\left(\frac{\ln x}{x}\right)\right) + C$

12. $\int \frac{x(x-1)}{(x^2+1)(x+1)\sqrt{x^3+x^2+x}}$

$= \frac{1}{2}\log\left|\frac{\sqrt{x+\frac{1}{x}+1}-1}{\sqrt{x+\frac{1}{x}+1}+1}\right| - A + c.$

Then the value of A is equal to

(a) $\cos^{-1}\sqrt{1+\frac{1}{x}}$ (b) $\tan^{-1}\sqrt{x+\frac{1}{x}+1}$

(c) $\cot^{-1}\sqrt{x+\frac{1}{x}}$ (d) $\sin^{-1}\sqrt{x+\frac{1}{x}+1}$

Answers Key

Single Correct Answer Type

1. (b) **2.** (a) **3.** (c) **4.** (c) **5.** (a)
6. (b) **7.** (c) **8.** (b) **9.** (c) **10.** (b)
11. (b) **12.** (b)

Cengage Learning India Pvt. Ltd.

DPP 7.4

Integration of Rational Functions

Subjective Type

1. Evaluate: $\int \frac{(x-1)^2}{x^4+x^2+1}dx$

2. Evaluate: $\int \frac{x^2-4}{x^4+9x^2+16}dx$

Single Correct Answer Type

3. $\int \frac{dx^3}{x^3(x^n+1)}$ equals

(a) $\frac{3}{n}\ln\left(\frac{x^n}{x^n+1}\right)$ (b) $\frac{1}{n}\ln\left(\frac{x^n}{x^n+1}\right)$

(c) $\frac{3}{n}\ln\left(\frac{x^n+1}{x^n}\right)$ (d) $3n\ln\left(\frac{x^{n+1}}{x^n}\right)$

4. $\int \frac{(x+1)^2\,dx}{x(x^2+1)}$ is equal to

(a) $\log_e x + c$ (b) $\log_e x + 2\tan^{-1}x + c$

(c) $\log_e \frac{1}{x^2+1} + c$ (d) $\log_e\{x(x^2+1)\} + c$

5. $\int \frac{x^3-x}{1+x^6}dx$ is equal to

(a) $\frac{1}{6}\log\frac{x^4-x^2+1}{(1+x^2)^2}+C$ (b) $\frac{1}{6}\tan^{-1}\frac{(x^2+1)^2}{2}+C$

(c) $\log\frac{x^4-x^2+1}{(1+x^2)^2}+C$ (d) $\tan^{-1}\frac{(x^2+1)^2}{2}+C$

6. $\int \frac{(x^3-1)}{(x^4+1)(x+1)}dx$ is

(a) $\frac{1}{4}\ln(1+x^4)+\frac{1}{3}\ln(1+x^3)+c$

(b) $\frac{1}{4}\ln(1+x^4)-\frac{1}{3}\ln(1+x^3)+c$

(c) $\frac{1}{4}\ln(1+x^4)-\ln(1+x)+c$

(d) $\frac{1}{4}\ln(1+x^4)+\ln(1+x)+c$

7. The value of $\int \frac{\cos^3 x}{\sin^2 x+\sin x}dx$ is equal to

(a) $\log_e|\sin x| + \sin x + C$
(b) $\log_e|\sin x| - \sin x + C$
(c) $-\log_e|\sin x| - \sin x + C$
(d) $-\log_e|\sin x| + \sin x + C$

8. $\int \frac{x^3-1}{x^3+x}dx$ is equal to

(a) $x - \log_e|x| + \log_e(x^2+1) - \tan^{-1}x + C$

(b) $x - \log_e|x| + \frac{1}{2}\log_e(x^2+1) - \tan^{-1}x + C$

(c) $x + \log_e|x| + \frac{1}{2}\log_e(x^2+1) + \tan^{-1}x + C$

(d) none of these

Comprehension Type

For Questions 9 and 10

Let $f(x) = \int \frac{dx}{e^x+8e^{-x}+4e^{-3x}}$, $g(x) = \int \frac{dx}{e^{3x}+8e^x+4e^{-x}}$.

9. $\int (f(x) - 2g(x))dx$

(a) $\frac{1}{2}\log\left|\frac{e^x+2e^{-x}-2}{e^x+2e^{-x}+2}\right|+C$

(b) $\frac{1}{4\sqrt{3}}\log\left|\frac{e^x-2e^{-x}-2\sqrt{3}}{e^x+2e^{-x}+2\sqrt{3}}\right|+C$

(c) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{e^x-2e^{-x}}{2\sqrt{3}}\right)+C$

(d) $\frac{1}{2}\tan^{-1}\left(\frac{e^x+2e^{-x}}{2}\right)+C$

Cengage Learning India Pvt. Ltd.

10. $\int (f(x) + 2g(x))dx$

(a) $\frac{1}{4\sqrt{3}}\log\left|\frac{e^x - 2e^{-x} - 2\sqrt{3}}{e^x + 2e^{-x} + 2\sqrt{3}}\right| + C$

(b) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{e^x - 2e^{-x}}{2\sqrt{3}}\right) + C$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{e^x + 2e^{-x}}{2}\right) + C$

(d) $\frac{1}{4}\log\left|\frac{e^x + 2e^{-x} - 2}{e^x + 2e^{-x} + 2}\right| + C$

Answers Key

Subjective Type

1. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{x - 1/x}{\sqrt{3}} - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$

2. 5

Single Correct Answer Type

3. (a) **4.** (b) **5.** (a) **6.** (c) **7.** (b)
8. (b)

Comprehension Type

9. (d) **10.** (b)

Cengage Learning India Pvt. Ltd.

DPP 7.5

Integration Using by Parts

Subjective Type

1. Evaluate: $\int \cos 2x \log_e (1 + \tan x) dx$
2. Evaluate: $\int \sin 4x \cdot e^{\tan^2 x}\, dx$
3. Evaluate: $\int \frac{\log_e(1+\sin^2 x)}{\cos^2 x} dx$
4. Evaluate: $\int \frac{x \log_e x}{(x^2-1)^{3/2}}\, dx$

Single Correct Answer Type

5. $\int \frac{(2+\sec x)\sec x}{(1+2\sec x)^2} dx =$

   (a) $\frac{1}{2\,\text{cosec}\, x + \cot x} + C$

   (b) $2\,\text{cosec}\, x + \cot x + C$

   (c) $\frac{1}{2\,\text{cosec}\, x - \cot x} + C$

   (d) $2\,\text{cosec}\, x - \cot x + C$

6. If $\int f(x)dx = g(x)$ and $f^{-1}(x)$ is differentiable, then $\int f^{-1}(x)\, dx$ is equal to

   (a) $g^{-1}(x) + C$

   (b) $xf^{-1} + C$

   (c) $xf^{-1}(x) - g(f^{-1}(x)) + C$

   (d) $f^{-1}(x) + C$

7. $\int \frac{e^{\cot x}}{\sin^2 x}(2 \ln \text{cosec}\, x + \sin 2x)dx$

   (a) $2e^{\cot x} \ln|\sin x| + c$ (b) $2e^{\tan x} \ln|\sin x| + c$

   (c) $2e^{\cot x} \ln|\cos x| + c$ (d) $2e^{\tan x} \ln|\cos x| + c$

8. $\int \frac{1-7\cos^2 x}{\sin^7 x \cos^2 x} dx = \frac{f(x)}{(\sin x)^7} + C$, then $f(x)$ is equal to

   (a) $\sin x$ (b) $\cos x$

   (c) $\tan x$ (d) $\cot x$

9. If $f(x) = \int e^x \left(\tan^{-1} x + \frac{2x}{(1+x^2)^2}\right) dx$, $f(0) = 0$ then the value of $f(1)$ is

   (a) $e\left(\frac{\pi}{4} - \frac{1}{2}\right) + 1$ (b) $e\left(\frac{\pi}{4} + \frac{1}{2}\right) + 1$

   (c) $e\left(\frac{\pi}{2} - \frac{1}{4}\right) + 1$ (d) $e^{-1}\left(\frac{\pi}{4} - \frac{1}{2}\right) + 1$

10. $\int \frac{e^x(x-1)(x-\ln x)}{x^2} dx$ is equal to

    (a) $e^x\left(\frac{x-\ln x}{x}\right) + c$ (b) $e^x\left(\frac{x-\ln x+1}{x}\right) + c$

    (c) $e^x\left(\frac{x-\ln x}{x^2}\right) + c$ (d) $e^x\left(\frac{x-\ln x-1}{x}\right) + c$

11. $\int (\sin(101x) \cdot \sin^{99} x) dx$ equals

    (a) $\frac{\sin(100x)(\sin x)^{100}}{100} + C$

    (b) $\frac{\cos(100x)(\sin x)^{100}}{100} + C$

    (c) $\frac{\cos(100x)(\cos x)^{100}}{100} + C$

    (d) $\frac{\cos(100x)(\cos x)^{100}}{100} + C$

Multiple Correct Answers Type

12. If $\int \frac{e^{4x}-1}{e^{2x}} \log\left(\frac{e^{2x}+1}{e^{2x}-1}\right) dx$

    $= \frac{t^2}{2}\log t - \frac{t^2}{4} - \frac{u^2}{2}\log u + \frac{u^2}{4} + C$, then

    (a) $u = e^x + e^{-x}$ (b) $u = e^x - e^{-x}$

    (c) $t = e^x + e^{-x}$ (d) $t = e^x - e^{-x}$

13. If $\int \frac{xe^x}{\sqrt{1+e^x}} dx = f(x)\sqrt{1+e^x} - 2\log g(x) + C$, then

    (a) $f(x) = x - 1$ (b) $g(x) = \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$

    (c) $g(x) = \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}$ (d) $f(x) = 2(x-2)$

Cengage Learning India Pvt. Ltd.

Answers Key

Subjective Type

1. $\frac{1}{2}[\sin 2x \log_e(1+\tan x) - x + \log|\sin x + \cos x|] + C$
2. $-2e^{\tan^2 x}\cos^4 x + C$
3. $\log_e(1+\sin^2 x)\cdot\tan x - 2x + \sqrt{2}\tan^{-1}(\sqrt{2}\tan x) + C$
4. $\frac{-\log_e x}{\sqrt{x^2-1}} + \sec^{-1}x + C$

Single Correct Answer Type

5. (a) **6.** (c) **7.** (a) **8.** (c) **9.** (a)
10. (d) **11.** (a)

Multiple Correct Answers Type

12. (b, c) **13.** (b, d)

Cengage Learning India Pvt. Ltd.

8 Definite Integration

DPP 8.1

Definite Integration: Definition

Single Correct Answer Type

1. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\ldots+\frac{1}{8n}\right]$ is equal to

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these

2. The value of $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sin\left\{\frac{(n+r)\pi}{4n}\right\}}\cdot\frac{\pi}{n}$ is equal to

(a) $2\ln(\sqrt{2}-1)$ (b) $4\ln(\sqrt{2}-1)$

(c) $4\ln(\sqrt{2}+1)$ (d) $\ln\sqrt{2}$

3. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{1/a\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}}{n^{a+1}}$ is equal to

(a) 1 (b) 2 (c) 1/2 (d) 4

4. Let a, b, c be non-zero real numbers such that $\int_0^3(3ax^2+2bx+c)\,dx=\int_1^3(3ax^2+2bx+c)\,dx$. Then

(a) $a+b+c=3$ (b) $a+b+c=1$

(c) $a+b+c=0$ (d) $a+b+c=2$

5. The number of solutions of the equation $\int_{-2}^{x}|\cos x|\,dx=0, 0<x<\frac{\pi}{2}$, is

(a) 0 (b) 1 (c) 2 (d) 4

6. $\int_0^1 e^{2\{x\}}e^{e^{\{x\}}}dx=$ (where $\{.\}$ denotes the fractional part function)

(a) $e^e(2e-1)$ (b) $e^e(e-1)$

(c) $e^{2e}(e-1)$ (d) none of these

7. Let $f(x)=\lim_{n\to\infty}\frac{\cos x}{1+(\tan^{-1}x)^n}$. Then the value of $\int_0^\infty f(x)dx$ is equal to

(a) cos (tan 1) (b) sin (tan 1)

(c) tan (tan 1) (d) none of these

8. The value of definite integral $\int_0^1\frac{dx}{\sqrt{(x+1)^3(3x+1)}}$ equals

(a) $\sqrt{2}-1$ (b) $\tan\frac{\pi}{12}$

(c) $\tan\frac{5\pi}{12}$ (d) none of these

Cengage Learning India Pvt. Ltd.

9. If $f(x)$ is continuous and $\int_0^9 f(x)\,dx = 4$, then the value of the integral $\int_0^3 x \cdot f(x^2)\,dx$ is

(a) 2 (b) 18 (c) 16 (d) 4

10. $\lim_{t \to 0} \int_0^{2\pi} \frac{|\sin(x+t) - \sin x|}{|t|} dx$ equals

(a) 2 (b) 4 (c) 1/4 (d) 1

Answers Key

Single Correct Answer Type

1. (a) **2.** (c) **3.** (a) **4.** (c) **5.** (a)

6. (b) **7.** (b) **8.** (a) **9.** (a) **10.** (b)

Cengage Learning India Pvt. Ltd.

DPP 8.2

Properties of Definite Integration, Definite Integration of Odd-Even and Periodic Functions

Single Correct Answer Type

1. The value of $I = \int_{-1}^{1} (1+x)^{1/2}(1-x)^{3/2}\,dx$ is

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) none of these

2. The value of $I = \int_{0}^{\pi} x(\sin^2(\sin x) + \cos^2(\cos x))\,dx$ is

(a) π^2 (b) $\frac{\pi^2}{2}$

(c) $\frac{\pi^2}{4}$ (d) none of these

3. $\int_{0}^{a} \ln(\cot a + \tan x)\,dx$, where $a \in \left(0, \frac{\pi}{2}\right)$, is

(a) $a \ln(\sin a)$ (b) $-a \ln(\sin a)$

(c) $-a \ln(\cos a)$ (d) none of these

4. If $f(x + f(y)) = f(x) + y\ \forall\ x, y \in R$ and $f(0) = 1$, then $\int_{0}^{10} f(10-x)\,dx$ is equal to

(a) 1 (b) 10

(c) $\int_{0}^{1} f(x)\,dx$ (d) $10\int_{0}^{1} f(x)\,dx$

5. Let $u = \int_{0}^{\pi/2} \cos\left(\frac{2\pi}{3}\sin^2 x\right)dx$ and $v = \int_{0}^{\pi/2} \cos\left(\frac{\pi}{3}\sin x\right)dx$, then the relation between u and v is

(a) $2u = v$ (b) $2u = 3v$ (c) $u = v$ (d) $u = 2v$

6. $\int_{0}^{100\pi} \left(\sum_{r=1}^{10} \tan rx\right)dx$ is equal to

(a) 0 (b) 100π (c) -50π (d) 50π

7. $\int_{0}^{\pi/2} \sin x \sin 2x \sin 3x \sin 4x\,dx =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{16}$ (d) $\frac{\pi}{32}$

8. $\int_{1}^{2013} [(x-1)(x-2)\ldots(x-2013)]dx$

(a) $(2013)^2$ (b) (2012)(2013)(2014)

(c) 2013! (d) 0

9. $f: [0, 5] \to R$, $y = f(x)$ such that $f''(x) = f''(5-x)\ \forall\ x \in [0, 5]$, $f'(0) = 1$ and $f'(5) = 7$, then the value of $\int_{1}^{4} f'(x)\,dx$ is

(a) 4 (b) 6 (c) 8 (d) 10

10. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{e^x \cdot \sec^2 x\,dx}{e^{2x}-1}$ is equal to

(a) 0 (b) 2 (c) e (d) $2e$

11. If f and g are two continuous functions being even and odd, respectively, then $\int_{-a}^{a} \frac{f(x)}{b^{g(x)}+1}dx$ is equal to (a being any non–zero number and b is positive real number, $b \neq 1$)

(a) independent of f

(b) independent of g

(c) independent of both f and g

(d) none of these

12. If $\int_{0}^{4\pi} \ln|13\sin x + 3\sqrt{3}\cos x|\,dx = k\pi \ln 7$, then the value of k is

(a) 2 (b) 4 (c) 8 (d) 16

13. f is a real valued function from R to R such that $f(x) + f(-x) = 2$, then $\int_{1-x}^{1+x} f^{-1}(t)\,dt =$

(a) -1 (b) 0

(c) 1 (d) none of these

Cengage Learning India Pvt. Ltd.

Multiple Correct Answers Type

14. If $A(x + y) = A(x)A(y)\ \forall x, y$ and $A(0) \neq 0$ and $B(x) = \dfrac{A(x)}{1+(A(x))^2}$, then

(a) $\int_{-2010}^{2010} B(x)dx = \int_{0}^{2011} B(x)dx$

(b) $\int_{-2010}^{2011} B(x)dx - \int_{0}^{2010} B(x)dx = \int_{0}^{2011} B(x)dx$

(c) $\int_{-2010}^{2011} B(x)dx = 0$

(d) $\int_{-2010}^{2010} (2B(-x) - B(x))dx = 2\int_{0}^{2010} B(x)dx$

15. A function f is defined by $f(x) = \int_{0}^{\pi} \cos t \cos(x - t)\, dt$, $0 \le x \le 2\pi$. Then which of the following hold(s) good?

(a) $f(x)$ is continuous but not differentiable in $(0, 2\pi)$.

(b) Maximum value of f is $\pi/2$.

(c) There exists atleast one $c \in (0, 2\pi)$ such that $f'(c) = 0$.

(d) Minimum value of f is $-\dfrac{\pi}{2}$.

Answers Key

Single Correct Answer Type

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)
6. (a)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (a)
11. (b)	**12.** (b)	**13.** (b)		

Multiple Correct Answers Type

14. (b, d) **15.** (b, c, d)

Cengage Learning India Pvt. Ltd.

DPP 8.3

Definite Integration Using Substitution

Single Correct Answer Type

1. If $\int_{\log 2}^{x} \frac{du}{(e^u-1)^{1/2}} = \frac{\pi}{6}$, then $e^x =$

(a) 1 (b) 2 (c) 4 (d) -1

2. The value of $\int_{e}^{\pi^2} [\log_\pi x] d(\log_e x)$ (where [.] denotes greatest integer function) is

(a) $2\log_e \pi$ (b) $\log_e \pi$
(c) 1 (d) 0

3. The value of $\frac{\int_0^1 \frac{dt}{\sqrt{1-t^4}}}{\int_0^1 \frac{1}{\sqrt{1+t^4}}dt}$ is

(a) 1 (b) 2 (c) $2\sqrt{3}$ (d) $\sqrt{2}$

4. Let a and b be two positive real numbers. Then the value of $\int_a^b \frac{e^{x/a}-e^{b/x}}{x}dx$ is

(a) 0 (b) ab (c) $1/ab$ (d) e^{ab}

5. The value of $\int_0^\infty \frac{\log x}{a^2+x^2}dx$ is

(a) $\frac{2\pi \log a}{a}$ (b) $\frac{\pi \log a}{2a}$
(c) $\pi \log a$ (d) 0

6. $\int_{1/3}^{3} \frac{1}{x}\log_e\left(\left|\frac{x+x^2-1}{x-x^2+1}\right|\right)dx$ is equal to

(a) $\frac{8}{3}$ (b) $-\frac{8}{3}$ (c) 0 (d) 3

7. If $I_n = \int_0^1 (1 + x + x^2 + \ldots + x^{n-1})(1 + 3x + 5x^2 + \ldots + (2n-3)x^{n-2} + (2n-1)x^{n-1})\,dx$, $n \in N$, then the value of $\sqrt{I_9}$ is

(a) 3 (b) 6
(c) 9 (d) 12

8. A function $f(x)$ satisfies $f(x) = f\left(\frac{c}{x}\right)$ for some real number $c(>1)$ and all positive number 'x'. If $\int_1^{\sqrt{c}} \frac{f(x)}{x}dx = 3$, then $\int_1^c \frac{f(x)}{x}dx$ is

(a) 4 (b) 6 (c) 8 (d) 9

9. Let $I_1 = \int_0^\infty \frac{x^2\sqrt{x}}{(1+x)^6}dx$, $I_2 = \int_0^\infty \frac{x\sqrt{x}}{(1+x)^6}dx$, then

(a) $I_1 = 2I_2$ (b) $I_2 = 2I_1$
(c) $I_1 = I_2^2$ (d) $I_1 = I_2$

Multiple Correct Answers Type

10. If $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ then

(a) $\int_0^\infty e^{-2x^2}dx = \frac{\sqrt{\pi}}{2\sqrt{2}}$ (b) $\int_0^\infty xe^{-x^2}dx = \frac{1}{2}$

(c) $\int_0^\infty x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{4}$ (d) $\int_0^\infty x^2e^{-x^2}dx = \frac{\pi}{4}$

11. Let $f(x) = \int_0^x \frac{e^t}{t}dt\,(x>0)$;

then $e^{-a}[f(x+a) - f(1+a)] =$

(a) $\int_0^x \frac{e^t}{(t+a)}dt$ (b) $\int_1^x \frac{e^t}{t+a}dt$

(c) $e^{-a}\int_{1+a}^{x+a} \frac{e^t}{t}dt$ (d) $\int_0^x \frac{e^{t-a}}{(t+a)}dt$

Cengage Learning India Pvt. Ltd.

Answers Key

Single Correct Answer Type

1. (c) 2. (b) 3. (d) 4. (a) 5. (b)
6. (c) 7. (c) 8. (b) 9. (d)

Multiple Correct Answers Type

10. (a, b, c) 11. (b, c)

DPP 8.4

Leibnitz Rule

Single Correct Answer Type

1. If $\int_0^{x^2(1+x)} f(t)\,dt = x$, then the value of $f(2)$ is

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

2. If $f(x) = \int_0^x \log_{0.5}\left(\frac{2t-8}{t-2}\right)dt$, then the interval in which $f(x)$ is increasing is

(a) $(-\infty, 2) \cup (6, \infty)$
(b) $(4, 6)$
(c) $(-\infty, 2) \cup (4, \infty)$
(d) $(2,6)$

3. If a, b and c are real numbers, then the value of $\lim_{t\to 0^+} \log_e\left(\frac{1}{t}\int_0^t (1 + a\sin bx)^{c/x}\,dx\right)$ equals

(a) abc (b) $\frac{ab}{c}$

(c) $\frac{bc}{a}$ (d) $\frac{ca}{b}$

4. If $f(x) = \int_2^x \frac{dt}{1+t^4}$, then

(a) $f(3) < \frac{1}{17}$ (b) $f(3) > \frac{1}{17}$

(c) $f(3) = \frac{1}{17}$ (d) $f(3) > 1$

5. If $\int_0^x f(x)\sin t\,dt$ = constant, $0 < x < 2\pi$ and $f(\pi) = 2$, then the value of $f(\pi/2)$ is

(a) 3 (b) 2 (c) 4 (d) 8

6. A function f, continuous on the positive real axis, has the property that for all choices of $x > 0$ and $y > 0$, the integral $\int_x^{xy} f(t)\,dt$ is independent of x (and therefore depends only on y). If $f(2) = 2$, then $\int_1^e f(t)\,dt$ is equal to

(a) e (b) $4e$
(c) 4 (d) none of these

7. The maximum value of the integral $\int_{a-1}^{a+1} \frac{1}{1+x^4}dx$ is attained

(a) exactly at two values of a
(b) only at one value of a which is positive
(c) only at one value of a which is negative
(d) only at $a = 0$

8. $\lim_{x\to 0} \frac{\int_0^x \frac{t^2}{\sqrt{a+t}}dt}{x - \sin x} = 1 (a > 0)$. Then the value of a is

(a) 1/2 (c) 1/4 (c) 2 (d) 4

9. Let $f(x)$ be a differentiable non-decreasing function such that $\int_0^x (f(t))^3\,dt = \frac{1}{x^2}\left(\int_0^x f(t)\,dt\right)^3 \forall x \in R - \{0\}$ and $f(1) = 1$. If $\int_0^x f(t)\,dt = g(x)$ then $\frac{xg'(x)}{g(x)}$ is

(a) always equal to 1 (b) always equal to -2
(c) may be 1 or -2 (d) not independent of x

10. Let f be continuous and the function g is defined as $g(x) = \int_0^x \left(t^2 \cdot \int_1^t f(u)du\right)dt$ where $f(1) = 3$. Then the value of $g'(1) + g''(1)$ is

(a) 1 (b) 2 (c) 3 (d) 4

Multiple Correct Answers Type

11. Let $f(x) = \int_2^x f(t^2 - 3t + 4)\,dt$. Then

(a) $f(2) = 0$ (b) $f(-2) = 0$
(c) $f'(2) = 0$ (d) $f''(2) = 2$

12. If $\int_0^x f(t)\,dt = e^x - ae^{2x}\int_0^1 f(t)e^{-t}\,dt$, then

Cengage Learning India Pvt. Ltd.

(a) $a = \dfrac{1}{3-2e}$ (b) $f(x) = e^x - 2e^{2x}$

(c) $a = \dfrac{1}{e}$ (d) $f(x) = e^x - e^{-x}$

13. A function $f(x)$ satisfies

$$f(x) = \sin x + \int_0^x f'(t)\,(2\sin t - \sin^2 t)\,dt$$

Then

(a) $f\left(\dfrac{\pi}{6}\right) = 1$

(b) $g(x) = \int_0^x f(t)\,dt$ is increasing on $(0, \pi)$

(c) $f(0) = 0$

(d) $f(x)$ is increasing on $(0, \pi)$

Comprehension Type

For Questions 14 and 15

Consider the function

$$f(x) = \int_0^x (5\ln(1+t^2) - 10t\tan^{-1}t + 16\sin t)\,dt$$

14. $f(x)$ is

(a) negative for all $x \in (0, 1)$

(b) increasing for all $x \in (0, 1)$

(c) decreasing for all $x \in (0, 1)$

(d) non-monotonic function for $x \in (0, 1)$

15. Which is not true for $\int_0^x f(t)\,dt$?

(a) positive for all $x \in (0, 1)$

(b) increasing for all $x \in (0, 1)$

(c) non-monotonic for all $x \in (0, 1)$

(d) none of these

Answers Key

Single Correct Answer Type

1. (d) **2.** (b) **3.** (a) **4.** (a) **5.** (c)
6. (c) **7.** (d) **8.** (d) **9.** (a) **10.** (c)

Multiple Correct Answers Type

11. (a, c) **12.** (a, b) **13.** (a, b, c)

Comprehension Type

14. (b) **15.** (c)

Cengage Learning India Pvt. Ltd.

DPP 8.5

Miscellaneous Exercises

Subjective Type

1. Prove that $\int_0^1 \frac{dx}{1+x^n} > 1 - \frac{1}{n}$ for $n \in N$

Single Correct Answer Type

2. Let $I_n = \int_0^{\pi/2} (\sin x + \cos x)^n \, dx \; (n \geq 2)$. Then the value of n. $I_n - 2(n-1) I_{n-2}$ is

(a) 5 (b) 9 (c) 2 (d) 7

3. Let $I_n = \int_0^1 x^n \sqrt{1-x^2} \, dx$. Then $\lim_{n\to\infty} \frac{I_n}{I_{n-2}} =$

(a) 2 (b) 1 (c) -1 (d) -2

4. If $\int_0^\infty e^{-ax} \, dx = \frac{1}{a}$, then $\int_0^\infty x^n e^{-ax} \, dx$ is

(a) $\frac{(-1)^n n!}{a^{n+1}}$ (b) $\frac{(-1)^n (n-1)!}{a^n}$

(c) $\frac{n!}{a^{n+1}}$ (d) none of these

5. Let $n \geq 1$, $n \in Z$. The real number $a \in 0, 1$ that minimizes the integral $\int_0^1 |x^n - a^n| dx$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{3}$

6. Let f be a continuous functions satisfying $f'(\ln x) = \begin{cases} 1 & \text{for } 0 < x \leq 1 \\ x & \text{for } x > 1 \end{cases}$ and $f(0) = 0$. Then $f(x)$ can be defined as

(a) $f(x) = \begin{cases} 1 & \text{if } x \leq 1 \\ 1 - e^x & \text{if } x > 1 \end{cases}$

(b) $f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

(c) $f(x) = \begin{cases} x & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases}$

(d) $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

7. $\frac{626 \int_0^\infty e^{-x} \sin^{25} x \, dx}{\int_0^\infty e^{-x} \sin^{23} x \, dx}$ is equal to

(a) 300 (b) 625 (c) 600 (d) 1200

8. If $g(x)$ is the inverse of $f(x)$ and $f(x)$ has domain $x \in [1, 5]$, where $f(1) = 2$ and $f(5) = 10$ then the value of $\int_1^5 f(x) dx + \int_2^{10} g(y) dy$ is

(a) 72 (b) 56 (c) 36 (d) 48

9. If $f x = x + \sin x$, then $\int_\pi^{2\pi} f^{-1}(x) dx$ is equal to

(a) $\frac{3\pi^2}{2} - 2$ (b) $\frac{3\pi^2}{2} + 2$

(c) $3\pi^2$ (d) none of these

10. Given a real-valued function f which is monotonic and differentiable. Then $\int_{f(a)}^{f(b)} 2x \, (b - f^{-1}(x)) dx =$

(a) $\int_a^b (f^2(x) - 2f^2(a)) dx$

(b) $\int_a^b (2f^2(x) - f^2(a)) dx$

(c) $\int_a^b (f^2(x) - f^2(a)) dx$

(d) None of these

Cengage Learning India Pvt. Ltd.

11. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then

(a) $\frac{1}{3} < I < \frac{1}{\sqrt{8}}$ (b) $\frac{1}{4} < I < \frac{1}{3}$

(c) $\frac{1}{4} < I < 0$ (d) none of these

12. Consider the function $h(x) = \frac{g^2(x)}{2} + 3x^3 - 5$, where $g(x)$ is a continuous and differentiable function. It is given that $h(x)$ is a monotonically increasing function and $g(0) = 4$. Then which of the following is not true?

(a) $g^2(1) > 10$ (b) $h(5) > 3$

(c) $h\left(\frac{5}{2}\right) < 2$ (d) $g^2(-1) < 22$

Comprehension Type

For Questions 13–15

Let m, n be two positive real numbers and define $f(n) = \int_0^\infty x^{n-1} e^{-x} dx$ and $g(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$.

It is known that $f(n)$ for $n > 0$ is finite and $g(m, n) = g(n, m)$ for $m, n > 0$.

13. $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx =$

(a) $g(m, n)$ (b) $g(m-1, n)$

(c) $g(m-1, n-1)$ (d) $g(m, n-1)$

14. $\int_0^1 x^m \left(\log_e \frac{1}{x}\right)^n dx =$

(a) $\frac{f(n+1)}{(m+1)^n}$ (b) $\frac{f(n)}{(m+1)^{n+1}}$

(c) $\frac{f(n+1)}{(m+1)^{n+1}}$ (d) $g(m+1, n+1)$

15. $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx =$

(a) $g(n, m)$ (b) $g(m-1, n+1)$

(c) $g(m-1, n-1)$ (d) $g(m+1, n-1)$

Answers Key

Single Correct Answer Type

2. (c) **3.** (b) **4.** (c) **5.** (a) **6.** (d)
7. (c) **8.** (d) **9.** (b) **10.** (c) **11.** (a)
12. (c)

Comprehension Type

13. (a) **14.** (c) **15.** (a)

Cengage Learning India Pvt. Ltd.

9 Area

DPP 9.1

Area Bounded by Curves (Elementary)

Single Correct Answer Type

1. The area enclosed between the curves $y = \sin^2 x$ and $y = \cos^2 x$ in the interval $0 \le x \le \pi$ is

(a) 2 sq unit (b) $\frac{1}{2}$ sq unit

(c) 1 sq unit (d) None of these

2. The area of the region(s) enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is

(a) 1/3 (b) 2/3 (c) 1/6 (d) 1

3. The area of the region bounded by $y = e^x$, $y = e^{-x}$, $x = 0$ and $x = 1$ is

(a) $e + \frac{1}{e}$ (b) $\log(4/e)$

(c) $4 \log(4/e)$ (d) $e + \frac{1}{e} - 2$

4. The area bounded by the curve $y = |\cos^{-1}(\sin x)| - |\sin^{-1}(\cos x)|$ and x-axis from $\frac{3\pi}{2} \le x \le 2\pi$ is

(a) π^2 sq. units (b) $\pi^2/4$ sq. units

(c) $\pi^2/2$ sq. units (d) none of these

5. If $(a, 0)$, $a > 0$, is the point where the curve $y = \sin 2x - \sqrt{3}\sin x$ cuts the x-axis first, A is the area bounded by this part of the curve, the origin and the positive x-axis. Then

(a) $4A + 8\cos a = 7$ (b) $4A + 8\sin a = 7$

(c) $4A - 8\sin a = 7$ (d) $4A - 8\cos a = 7$

6. The area in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$ is

(a) $\frac{(\pi^3 - 8)}{4}$ (b) $\frac{\pi^3}{4}$

(c) $\frac{(\pi^3 - 16)}{4}$ (d) $\frac{(\pi^3 - 8)}{2}$

7. The area bounded by the curves $y = \cos^{-1} x$; $y = \sin^{-1} x$ and $y = -\pi x^3$, where $-1 \le x \le 1$, is

(a) $\frac{3\pi}{2} + 1 - \sqrt{2}$ sq. units

(b) $\frac{3\pi}{4} + 1 + \sqrt{2}$ sq. units

(c) $\frac{3\pi}{4} + 2 - \sqrt{2}$ sq. units

(d) $\frac{3\pi}{4} + 1 - \sqrt{2}$ sq. units

8. The area bounded by the curve $y = \sin^2 x - 2\sin x$ and the x-axis, where $x \in [0, 2\pi]$, is

(a) 4 sq. units (b) 8 sq. units

(c) 16 sq. units (d) 20 sq. units

9. Consider the functions $f(x)$ and $g(x)$, both defined from $R \to R$ and are defined as $f(x) = 2x - x^2$ and $g(x) = x^n$ where $n \in N$. If the area between $f(x)$ and $g(x)$ is 1/2, then the value of n is

(a) 5 (b) 6 (c) 7 (d) 8

Cengage Learning India Pvt. Ltd.

10. Let a function $f(x)$ is defined in $[-2, 2]$ as

$$f(x)=\begin{cases}\{x\}, & -2\le x<-1\\ |\text{sgn } x|, & -1\le x\le 1\\ \{-x\}, & 1<x\le 2\end{cases}$$, where $\{x\}$ and sgn x denotes fractional part and signum functions, respectively. Then the area bounded by the graph of $f(x)$ and x-axis is

(a) 2 sq. units (b) 3 sq. units
(c) 4 sq. units (d) 5 sq. units

11. The area bounded by $y = x^2 + 2$ and $y = 2|x| - \cos \pi x$ is equal to

(a) 2/3 (b) 8/3
(c) 4/3 (d) 1/3

12. Area bounded by $f(x) = \dfrac{x^2-1}{x^2+1}$ and the line $y = 1$ is

(a) π sq. units (b) 2π sq. units
(c) $\pi/2$ sq. units (d) none of these

13. The area bounded by the curve $y = x\,e^{-x}$, $y = 0$ and $x = c$, where c is the x-coordinate of the curve's inflection point, is

(a) $1 - 3e^{-2}$ (b) $1 - 2e^{-2}$
(c) $1 - e^{-2}$ (d) none of these

14. Area of the region bounded by the curve $y = \dfrac{16-x^2}{4}$ and $y = \sec^{-1}[-\sin^2 x]$ (where [.] denotes the greatest integer function) is

(a) $\frac{1}{3}(4-\pi)^{3/2}$ (b) $8(4-\pi)^{3/2}$
(c) $\frac{8}{3}(4-\pi)^{3/2}$ (d) $\frac{8}{3}(4-\pi)^{1/2}$

Multiple Correct Answers Type

15. If the area bounded between x-axis and the graph of $y = 6x - 3x^2$ between the ordinates $x = 1$ and $x = a$ is 19 square units, then a can take the value

(a) one value in (2, 3) (b) one value in (–2,–1)
(c) one value in (–1, 0) (d) one value in (3, 4)

16. Which of the following is the possible value/values of c for which the area of the figure bounded by the curves $y = \sin 2x$, the straight lines $x = \pi/6$, $x = c$ and the abscissa axis is equal to 1/2?

(a) $-\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{6}$ (d) none of these

17. Area of the region bounded by the curve $y = \tan x$ and lines $y = 0$ and $x = 1$ is

(a) $\int_0^1 \tan(1-x)\,dx$ (b) $\tan 1 - \int_0^{\tan 1} \tan^{-1} y\,dy$
(c) $\int_0^{\tan 1} \tan^{-1} y\,dy$ (d) $\int_0^1 \tan^{-1} x\,dx$

Answers Key

Single Correct Answer Type

1. (c)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)
6. (a)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (b)
11. (b)	**12.** (b)	**13.** (a)	**14.** (c)	

Multiple Correct Answers Type

15. (b, d) **16.** (a, b) **17.** (a, b)

Cengage Learning India Pvt. Ltd.

DPP 9.2

Area Bounded by Curves (Typical)

Single Correct Answer Type

1. Suppose $y = f(x)$ and $y = g(x)$ are two functions whose graphs intersect at the three points (0, 4), (2, 2) and (4, 0) with $f(x) > g(x)$ for $0 < x < 2$ and $f(x) < g(x)$ for $2 < x < 4$. If $\int_0^4 [f(x) - g(x)]dx = 10$ and $\int_2^4 [g(x) - f(x)]dx = 5$, the area between two curves for $0 < x < 2$, is
(a) 5 (b) 10 (c) 15 (d) 20

2. The ratio of the areas of two regions of the curve $C_1 \equiv 4x^2 + \pi^2 y^2 = 4\pi^2$ divided by the curve $C_2 \equiv y = -\left(\text{sgn}\left(x - \frac{\pi}{2}\right)\right)\cos x$ (where sgn (x) = signum (x)) is
(a) $\frac{\pi^2 - 2}{\pi^2 + 2}$ (b) $\frac{\pi^2 + 2}{\pi^2 - 2\sqrt{2}}$
(c) $\frac{\pi^2 + 6}{\pi^2 + 3\sqrt{2}}$ (d) $\frac{\pi^2 - 1}{\pi^2 - \sqrt{2}}$

3. The area bounded by the curves
$x\sqrt{3} + y = 2\log_e(x - y\sqrt{3}) - 2\log_e 2$, $y = \sqrt{3}x$,
$y = -\frac{1}{\sqrt{3}}x + 2$. is
(a) $2\log_e 2$ sq. units (b) $2\log_e 2 + 1$ sq. units
(c) $2\log_e 2 - 1$ sq. units (d) $4\log_e 2 - 1$ sq.units

4. Area of region bounded by the curves $y = \frac{4 - x^2}{4 + x^2}$, $25y^2 = 9x$ and $y = \frac{3}{5}|x| - \frac{6}{5}$ which contains (1, 0) point in its interior is
(a) $\left\{\pi - 4\tan^{-1}\frac{1}{2} + \frac{1}{10}\right\}$ sq. units
(b) $\left\{\pi - 2\tan^{-1}\frac{1}{2} - \frac{1}{5}\right\}$ sq. units
(c) $\left\{\pi + 4\tan^{-1}\frac{1}{2} - \frac{1}{5}\right\}$ sq. units
(d) none of these

5. Area bounded by the min. $\{|x|, |y|\} = 1$ and the max. $\{|x|, |y|\} = 2$ is
(a) 4 (b) 8 (c) 16 (d) 9

6. Consider $f(x) = \begin{cases} \cos x & 0 \le x < \frac{\pi}{2} \\ \left(\frac{\pi}{2} - x\right)^2 & \frac{\pi}{2} \le x < \pi \end{cases}$ such that f is periodic with period π. Then which of the following is not true?
(a) The range of f is $\left[0, \frac{\pi^2}{4}\right)$.
(b) f is discontinuous for infinite values of x.
(c) The area bounded by $y = f(x)$ and the X-axis from $x = 0$ to $x = n\pi$ is $n\left(1 + \frac{\pi^3}{24}\right)$ for a given $n \in N$.
(d) None of these

7. The area made by curve $f(x) = [x] + \sqrt{x - [x]}$ and x-axis when $0 \le x \le n$ $(n \in N)$ is equal to {where $[x]$ is greatest integer function}
(a) $\frac{2n}{3} + \frac{n(n+1)}{2}$ (b) $\frac{n}{3} + \frac{n(n+1)}{2}$
(c) $\frac{2n}{3} + \frac{n(n-1)}{2}$ (d) $\frac{n}{3} + \frac{n(n-1)}{2}$

8. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \le 100\}$ and $B = \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then the area of the region $A \cap B$ is
(a) 10π (b) 100 (c) 100π (d) 50π

9. Let R be the region containing the point (x, y) on the X-Y plane, satisfying $2 \le |x + 3y| + |x - y| \le 4$. Then the area of this region is
(a) 5 sq. units (b) 6 sq. units
(c) 7 sq. units (d) 8 sq. units

10. If $f(x) = \begin{cases} \sqrt{\{x\}} & \text{for } x \notin Z \\ 1 & \text{for } x \in Z \end{cases}$ and $g(x) = \{x\}^2$ where $\{.\}$ denotes fractional part of x then area bounded by $f(x)$ and $g(x)$ for $x \in 0, 6$ is
(a) $\frac{2}{3}$ (b) 2 (c) $\frac{10}{3}$ (d) 6

Cengage Learning India Pvt. Ltd.

11. Let S is the region of points which satisfies $y^2 < 16x$, $x < 4$ and $\frac{xy(x^2-3x+2)}{x^2-7x+12} > 0$. Its area is

(a) $\frac{8}{3}$ (b) $\frac{64}{3}$ (c) $\frac{32}{3}$ (d) none of these

12. The area of the region $\{(x, y): x^2 + y^2 \le 5, ||x| - |y|| \ge 1$ is

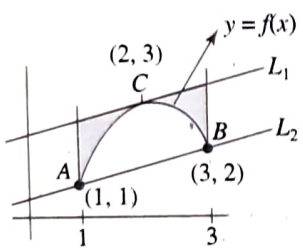
(a) $4\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 4$

(b) $5\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 4$

(c) $3\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 4$

(d) $2\left(\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) - 1$

13. The following figure shows the graph of a continuous function $y = f(x)$ on the interval [1, 3]. The points A, B, C have coordinates (1, 1), (3, 2), (2, 3), respectively, and the lines L_1 and L_2 are parallel, with L_1 being tangent to the curve at C. If the area under the graph of $y = f(x)$ from $x = 1$ to $x = 3$ is 4 square units, then the area of the shaded region is



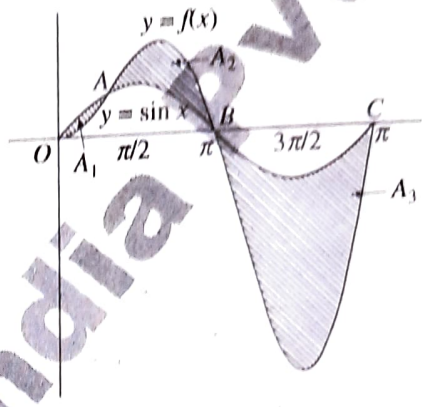
(a) 1 (b) 2 (c) 3 (d) 4

Comprehension Type

For Questions 14 and 15

In the following figure, the graphs of two functions $y = f(x)$ and $y = \sin x$ are given. They intersect at origin, $A(a, f(a))$, $B(\pi, 0)$ and $C(2\pi, 0)$. A_i $(i = 1, 2, 3)$ is the area bounded by the curves as shown in the figure, respectively, for $x \in (0, a)$, $x \in (a, \pi)$, $x \in (\pi, 2\pi)$.

If $A_1 = 1 + (a - 1)\cos a - \sin a$, then



14. The function $f(x)$ is

(a) $x^2 \sin x$ (b) $x \sin x$

(c) $2x \sin x$ (d) $x^3 \sin x$

15. The value of A_2 is

(a) $(\pi - 1)$ units^2 (b) $(\pi/2 - 1)$ units^2

(c) $(\pi - \sin 1 - 1)$ units^2 (d) $\pi/2$ units^2

Answers Key

Single Correct Answer Type

1. (c)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (a)
6. (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)	**13.** (b)		

Comprehension Type

14. (b) **15.** (c)

Cengage Learning India Pvt. Ltd.

10 Differential Equations

DPP 10.1

Forming Differential Equation, Differential Equations with Variable Separable, Homogeneous Differential Equations

Single Correct Answer Type

1. The order of the differential equation whose general solution is $y = c_1 \cos 2x + c_2 \cos^2 x + c_3 \sin^2 x + c_4$ is
   (a) 2 (b) 4
   (c) 3 (d) None of these

2. Order of the differential equation whose general solution is $y = \frac{ax}{bx + c}$, where a, b, c are arbitrary constants is
   (a) 1 (b) 2
   (c) 3 (d) none of these

3. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x$, then
   (a) $p > q$ (b) $\frac{p}{q} = \frac{1}{2}$
   (c) $p = q$ (d) $p < q$

4. If m and n are the order and degree of the differential equation $(y'')^5 + 4 \cdot \frac{(y'')^3}{y'''} + y''' = \sin x$, then
   (a) $m = 3, n = 5$ (b) $m = 3, n = 1$
   (c) $m = 3, n = 3$ (d) $m = 3, n = 2$

5. If the differential equation $\frac{dx}{3y + f} + \frac{dy}{px + g} = 0$ represents a family of circle, then $p =$
   (a) g (b) f (c) 4 (d) 3

6. The general solution of $\frac{dy}{dx} = 1 - x^2 - y^2 + x^2y^2$ is
   (a) $2 \sin^{-1} y = x\sqrt{1 - y^2} + c$
   (b) $\sin^{-1} y = \frac{1}{2} \sin^{-1} x + c$
   (c) $\cos^{-1} y = x \cos^{-1} x + c$
   (d) $\frac{1}{2} \log\left(\frac{1+y}{1-y}\right) = x - \frac{x^3}{3} + C$

7. The solution of the differential equation $e^{-x}(y + 1)dy + (\cos^2 x - \sin 2x)y\,(dx) = 0$ subjected to the condition that $y = 1$ when $x = 0$ is
   (a) $(y + 1) + e^x \cos^2 x = 2$
   (b) $y + \log y = e^x \cos^2 x$
   (c) $\log (y + 1) + e^x \cos^2 x = 1$
   (d) $y + \log y + e^x \cos^2 x = 2$

8. The solution of the differential equation $(x^2 - xy^2)\frac{dy}{dx} + y^2 + xy^2 = 0$ is
   (a) $\log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$ (b) $\log\left(\frac{y}{x}\right) = \frac{1}{x} + \frac{1}{y} + c$
   (c) $\log (xy) = \frac{1}{x} + \frac{1}{y} + c$ (d) $\log (xy) + \frac{1}{x} + \frac{1}{y} = c$

Cengage Learning India Pvt. Ltd

9. The family of curves passing through (0, 0) and satisfying the differential equation $\frac{y_2}{y_1} = 1$ (where $y_n = d^n y/dx^n$) is

(a) $y = k$ (b) $y = kx$

(c) $y = k(e^x + 1)$ (d) $y = k(e^x - 1)$

10. The solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ is

(a) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$

(b) $2\tan^{-1}\left(\frac{x}{y}\right) + \ln x + C = 0$

(c) $\ln\left(y + \sqrt{x^2 + y^2}\right) + \ln y + C = 0$

(d) $\ln\left(x + \sqrt{x^2 + y^2}\right) + C = 0$

11. The solution of differential equation $(1 - xy + x^2y^2)dx = x^2dy$ is

(a) $\tan xy = \log|cx|$ (b) $\tan(y/x) = \tan\log|cx|$

(c) $xy = \tan\log|cx|$ (d) None of these

Multiple Correct Answers Type

12. The differential equation for the family of curves $y = c\sin x$ can be given by

(a) $\left(\frac{dy}{dx}\right)^2 = y^2\cot^2 x$

(b) $\left(\frac{dy}{dx}\right)^2 - \left(\sec x\frac{dy}{dx}\right)^2 + y^2 = 0$

(c) $\left(\frac{dy}{dx}\right)^2 = \tan^2 x$

(d) $\frac{dy}{dx} = y\cot x$

13. If the solution of the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0$ given that for $t = 0$, $x = 0$ and $\frac{dx}{dt} = 12$ is in the form $x = Ae^{-3t} + Be^{-t}$, then

(a) $A + B = 0$ (b) $A + B = 12$

(c) $|AB| = 36$ (d) $|AB| = 49$

14. The solution of $\left(\frac{dy}{dx}\right)^2 - 2\left(x + \frac{1}{4x}\right)\frac{dy}{dx} + 1 = 0$

(a) $y = x^2 + c$ (b) $y = \frac{1}{2}\ln(x) + c, x > 0$

(c) $y = \frac{x}{2} + c$ (d) $y = \frac{x^2}{2} + c$

15. $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ has the solution

(a) $y - \frac{c}{1 + \cos x} = 0$ (b) $y = \frac{c}{1 - \cos x}$

(c) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$ (d) $x = 2\cos^{-1}\left(\frac{c}{2y}\right)$

Answers Key

Single Correct Answer Type

1. (a) **2.** (b) **3.** (d) **4.** (d) **5.** (d)
6. (d) **7.** (d) **8.** (a) **9.** (d) **10.** (a)
11. (c)

Multiple Correct Answers Type

12. (a, b, d) **13.** (a, c) **14.** (a, b) **15.** (a, b)

Cengage Learning India Pvt. Ltd.

DPP 10.2

Linear Differential Equations

Single Correct Answer Type

1. If $y(t)$ satisfies the differential equation $y'(t) + 2y(t) = 2e^{-2t}$, $y(0) = 2$ then $y(1)$ equals

(a) $\frac{3}{e}$ (b) $\frac{3}{e^2}$ (c) $\frac{4}{e}$ (d) $\frac{4}{e^2}$

2. If $\frac{dy}{dx} - y \log_e 2 = 2^{\sin x}(\cos x - 1)\log_e 2$, then $y =$

(a) $2^{\sin x} + c2^x$ (b) $2^{\cos x} + c2^x$

(c) $2^{\sin x} + c2^{-x}$ (d) $2^{\cos x} + c2^{-x}$

3. If $ye^y\,dx = (y^3 + 2xe^y)dy$, $y(0) = 1$, then the value of x when $y = 0$ is

(a) -1 (b) 0 (c) 1 (d) 2

4. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx} - f(x)y = 0$, then a solution of the differential equation $\frac{dy}{dx} + f(x)y = r(x)$ is

(a) $y = \frac{1}{y_1(x)}\int r(x)\, y_1(x)\,dx + \frac{c}{y_1(x)}$

(b) $y = y_1(x)\int \frac{r(x)}{y_1(x)}dx + c$

(c) $y = \int r(x)y_1(x)\,dx + c$

(d) none of these

5. The general solution of $x\left(\frac{dy}{dx}\right) + (\log x)y = x^{-\frac{1}{2}\log x}$ is

(a) $y = x^{1-\frac{1}{2}\log x} + cx^{-\frac{1}{2}\log x}$

(b) $y \cdot x^{\frac{1}{2}\log x} = x^{\frac{1}{2}\log x} + c$

(c) $y = e^{\frac{(\log x)^2}{2}}(x + c)$

(d) $y = e^{\frac{1}{2}(\log x)^2}(x^{1-\frac{1}{2}(\log x)} - x^{-\frac{1}{2}\log x}) + c$

6. The general solution of the differential equation $(1 + \tan y)(dx - dy) + 2xdy = 0$ is

(a) $x(\sin y + \cos y) = \sin y + ce^y$

(b) $x(\sin y + \cos y) = \sin y + ce^{-y}$

(c) $y(\sin x + \cos x) = \sin x + ce^x$

(d) None of these

7. Solution of differential equation $x^2y - x^3\frac{dy}{dx} = y^4 \cos x$ is

(a) $x^2y^{-3} = 2\sin x + c$ (b) $x^2y^{-3} = 3\cos x + c$

(c) $x^3y^{-3} = 3\sin x + c$ (d) $x^2y^3 = 3\sin x + c\,x^2y$

8. Suppose a solution of the differential equation $(xy^3 + x^2y^7)\frac{dy}{dx} = 1$ satisfies the initial conditions $y\left(\frac{1}{4}\right) = 1$. Then the value of $\frac{dy}{dx}$ when $y = -1$ is

(a) $-\frac{3}{20}$ (b) $-\frac{20}{3}$

(c) $-\frac{5}{16}$ (d) $-\frac{16}{5}$

9. The general solution of the differential equation $\frac{dy}{dx} = y\tan x - y^2\sec x$ is

(a) $\tan x = (c + \sec x)y$ (b) $\sec y = (c + \tan y)x$

(c) $\sec x = (c + \tan x)y$ (d) none of these

Multiple Correct Answers Type

10. Let $\frac{dy}{dx} + y = f(x)$ where y is a continuous function of x with $y(0) = 1$ and $f(x) = \begin{cases} e^{-x} & \text{if } 0 \le x \le 2 \\ e^{-2} & \text{if } x > 2 \end{cases}$. Which of the following hold(s) good?

(a) $y(1) = 2e^{-1}$ (b) $y'(1) = -e^{-1}$

(c) $y(3) = -2e^{-3}$ (d) $y'(3) = -2e^{-3}$

11. A differentiable function satisfies

$$f(x) = \int_0^x (f'(t)\cos t - \cos(t - x))\,dt.$$

Which of the following hold(s) good?

(a) $f(x)$ has a minimum value $1 - e$

(b) $f(x)$ has a maximum value $1 - e^{-1}$

Cengage Learning India Pvt. Ltd.

(c) $f''\left(\frac{\pi}{2}\right) = e$

(d) $f'(0) = 1$

12. $y = f(x)$ which has differential equation $y(2xy + e^x)dx - e^x dy = 0$ passing through the point (0, 1). Then which of the following is/are true about the function?

(a) $x = 1 + \sqrt{2}$ is point of local maxima

(b) $x = 1 - \sqrt{2}$ is point of local minima

(c) $\lim_{x \to \infty} f(x) = -\infty$

(d) $\lim_{x \to -\infty} f(x) = 0$

Answers Key

Single Correct Answer Type

1. (d) **2.** (a) **3.** (b) **4.** (a) **5.** (a)
6. (b) **7.** (c) **8.** (d) **9.** (c)

Multiple Correct Answers Type

10. (a, b, d) **11.** (a, b, c) **12.** (a, b, c, d)

Cengage Learning India Pvt. Ltd.

DPP 10.3

General Form of Variable Separation

Single Correct Answer Type

1. The solution of differential equation $x^2(x\,dy + y\,dx) = (xy-1)^2\,dx$ is (where c is an arbitrary constant)

(a) $xy - 1 = cx$ (b) $xy - 1 = cx^2$

(c) $\frac{1}{xy-1} = \frac{1}{x} + c$ (d) None of these

2. Solution of the differential $(x + 2y^3) = \frac{dx}{dy}y$ is

(a) $x = y^2(c + y^2)$ (b) $x = y(c - y^2)$

(c) $x = 2y(c - y^2)$ (d) $x = y(c + y^2)$

3. General solution of differential equation

$$x^2\left(x + y\frac{dy}{dx}\right) + \left(x\frac{dy}{dx} - y\right)\sqrt{x^2+y^2} = 0 \text{ is}$$

(a) $\frac{1}{\sqrt{x^2+y^2}} + \frac{y}{x} = c$ (b) $\sqrt{x^2+y^2} - \frac{y}{x} = c$

(c) $\sqrt{x^2+y^2} + \frac{y}{x} = c$ (d) $2\sqrt{x^2+y^2} + \frac{y}{x} = c$

4. Solution of the differential $y' = \frac{3yx^2}{x^3 + 2y^4}$ is

(a) $x^3y^{-1} = \frac{2}{3}y^3 + c$ (b) $x^2y^{-1} = \frac{2}{3}y^3 + c$

(c) $xy^{-1} = \frac{2}{3}y^3 + c$ (d) None of these

5. For $y > 0$ and $x \in R$, $ydx + y^2dy = xdy$ where $y = f(x)$. If $f(1) = 1$, then the value of $f(-3)$ is

(a) 1 (b) 2 (c) 3 (d) 4

6. An equation of the curve satisfying $x\,dy - y\,dx = \sqrt{x^2 - y^2}\,dx$ and $y(1) = 0$ is

(a) $y = x^2 \log|\sin x|$ (b) $y = x \sin(\log|x|)$

(c) $y^2 = x(x-1)^2$ (d) $y = 2x^2(x-1)$

7. The solution of $(y(1 + x^{-1}) + \sin y)dx + (x + \log x + x\cos y)dy = 0$ is

(a) $(1 + y^{-1}\sin y) + x^{-1}\log x = c$

(b) $(y + \sin y) + xy\log x = C$

(c) $xy + y\log x + x\sin y = C$

(d) none of these

8. The solution of $(1 + x)\frac{dy}{dx} + 1 = e^{x-y}$ is

(a) $e^y(x + 1) = c$ (b) $e^y(x + 1) = e^x + c$

(c) $e^y(x + 1) = ce^x$ (d) $(x + 1) = e^x + c$

9. The solution of differential equation

$$x\sec\left(\frac{y}{x}\right)(y\,dx + x\,dy) = y\,\text{cosec}\left(\frac{y}{x}\right)(x\,dy - y\,dx) \text{ is}$$

(a) $xy = c\,\text{cosec}\left(\frac{y}{x}\right)$ (b) $xy^2\sin\frac{y}{x} = c$

(c) $xy\,\text{cosec}\frac{y}{x} = c$ (d) $xy = c\sin\left(\frac{x}{y}\right)$

10. The general solution of the differential equation

$\sqrt{1 - x^2y^2}\,dx = y\,dx + x\,dy$ is

(a) $\sin(xy) = x + c$ (b) $\sin^{-1}(xy) + x = c$

(c) $\sin(x + c) = xy$ (d) $\sin(xy) + x = c$

11. The solution of $(1 + xy)y\,dx + (1 - xy)x\,dy = 0$ is

(a) $\frac{x}{y} + \frac{1}{xy} = k$ (b) $\log\left(\frac{x}{y}\right) = \frac{1}{xy} + k$

(c) $\frac{x}{y} + \frac{1}{xy} = k$ (d) $\log\left(\frac{x}{y}\right) = xy + k$

12. Solution of $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ is

(a) $e^{-2x}y^2 + 2\ln|y| = c$ (b) $e^{2x}y^2 - 2\ln|y| = c$

(c) $e^x + \ln|y| = c$ (d) none of these

Answers Key

Single Correct Answer Type

1. (c) **2.** (d) **3.** (c) **4.** (a) **5.** (c)
6. (b) **7.** (c) **8.** (b) **9.** (c) **10.** (c)
11. (c) **12.** (a)

Cengage Learning India Pvt. Ltd.

DPP 10.4

Applications of Differential Equations

Single Correct Answer Type

1. A population grows at the rate of 10% of the population per year. How long does it take for the population to double?
   (a) 2 log 10 years (b) 20 log 2 years
   (c) 10 log 2 years (d) 5 log 2 years
2. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x and y axes at A and B, respectively, such that $AP : PB = 1 : 3$. If $f(1) = 1$ then the curve passes through $\left(k, \frac{1}{8}\right)$ where k is
   (a) 1 (b) 2 (c) 3 (d) 4
3. A curve C with negative slope through the point (0,1) lies in the first quadrant. The tangent at any point P on it meets the x-axis at Q such that $PQ = 1$. Then the curve in parametric form is
   (a) $x = \cos\theta + \ln\tan(\theta/2), y = \sin\theta$
   (b) $x = -\cos\theta + \ln\tan(\theta/2), y = \sin\theta$
   (c) $x = -\cos\theta - \ln\tan(\theta/2), y = \sin\theta$
   (d) None of these
4. Tangent to a curve intersect y-axis at a point P. A line perpendicular to this tangent through P passes through point (1, 0). The differential equation of the curves is
   (a) $y\cdot\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$ (b) $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 1$
   (c) $y\cdot\frac{dx}{dy} + x = 1$ (d) None of these
5. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are given by
   (a) $x^n + n^2y = \text{const}$ (b) $ny^2 + x^2 = \text{const}$
   (c) $n^2x + y^n = \text{const}$ (d) $n^2x - y^n = \text{const}$

Multiple Correct Answers Type

6. Suppose a curve whose sub tangent is n times the abscissa of the point of contact and passes through the point (2, 3). Then
   (a) for $n = 1$, equation of the curve is $2y = 3x$
   (b) for $n = 1$, equation of the curve is $2y^2 = 9x$
   (c) for $n = 2$, equation of the curve is $2y = 3x$
   (d) for $n = 2$, equation of the curve is $2y^2 = 9x$
7. Let C be a curve such that the normal at any point P on it meets x-axis and y-axis at A and B respectively. If $BP : PA = 1 : 2$ (internally) and the curve passes through the point (0,4), then which of the following alternative(s) is/are correct?
   (a) The curves passes through $(\sqrt{10}, -6)$.
   (b) The equation of tangent at $(4, 4\sqrt{3})$ is $2x + \sqrt{3}y = 20$.
   (c) The differential equation for the curve is $yy' + 2x = 0$.
   (d) The curve represents a hyperbola.
8. The normal at a general point (a, b) on curve makes an angle θ with x-axis which satisfies $b(a^2\tan\theta - \cot\theta) = a(b^2 + 1)$. The equation of curve can be
   (a) $y = e^{x^2/2} + c$ (b) $\log ky^2 = x^2$
   (c) $y = ke^{x^2/2}$ (d) $x^2 - y^2 = k$

Comprehension Type

For Questions 9 and 10

Tangent is drawn at the point (x_i, y_i) on the curve $y = f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now again tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersects the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times i.e. $i = 1, 2, 3, \ldots, n$.

9. If $x_1, x_2, x_3, \ldots, x_n$ form an arithmetic progression with common difference equal to $\log_2 e$ and curve passes through (0, 2), then the equation of the curve passes through
   (a) (1, 4) (b) (5, 1/16)
   (c) (2, 1/2) (d) none of these
10. If $x_1, x_2, x_3, \ldots, x_n$ form a geometric progression with common ratio equal to 2 and the curve passes through (1, 2), then the curve is
    (a) circle (b) hyperbola
    (c) ellipse (d) parabola

Answers Key

Single Correct Answer Type

1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (b)

Multiple Correct Answers Type

6. (a, d) **7.** (a, d) **8.** (b, c, d)

Comprehension Type

9. (b) **10.** (b)

Cengage Learning India Pvt. Ltd.